

Chapter 251

P Charts

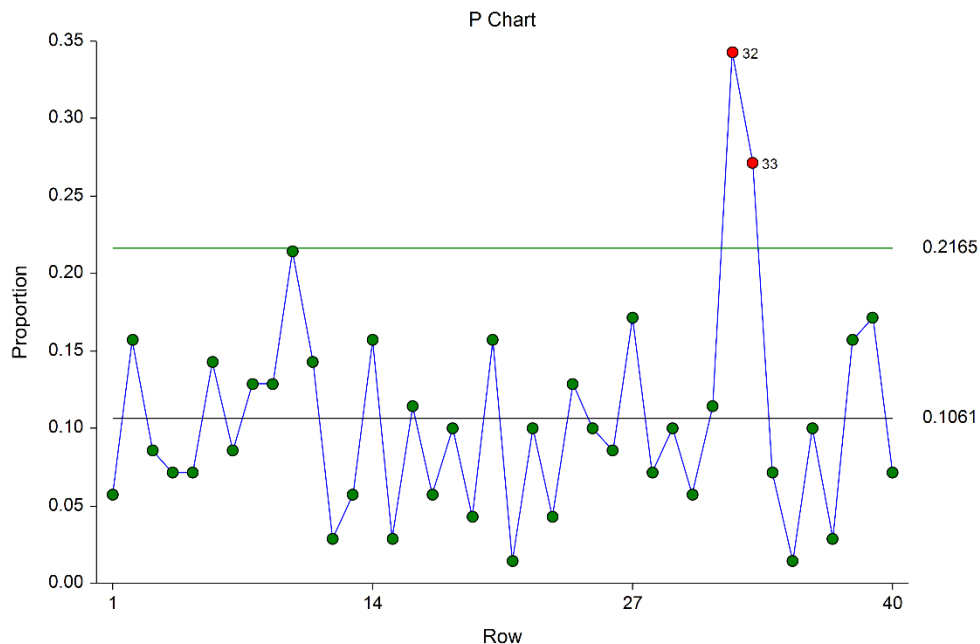
Introduction

This procedure generates the P control chart for the fraction of a sample that is nonconforming. The format of the control charts is fully customizable. This procedure permits the defining of stages. For the P chart, the value for P can be entered directly or estimated from the data, or a sub-set of the data. A list of out-of-control points can be produced in the output, if desired, and proportion nonconforming values may be stored to the spreadsheet.

P Control Charts

P charts are used to monitor the proportion of nonconforming units of a process based on samples taken from the process at given times (hours, shifts, days, weeks, months, etc.). Typically, an initial series of samples is used to estimate proportion nonconforming of a process. The estimated proportion is then used to produce control limits for the proportions. During this initial phase, the process should be in control. If points are out-of-control during the initial (estimation) phase, the assignable cause should be determined, and the sample should be removed from estimation.

Once the control limits have been established for the P chart, these limits may be used to monitor the proportion nonconforming of the process going forward. When a point is outside these established control limits it indicates that the proportion nonconforming of the process is out-of-control. An assignable cause is suspected whenever the control chart indicates an out-of-control process.



The P Chart versus the NP Chart

The NP chart is very similar to the P chart. Rather than focusing on the proportion of nonconforming units, as does the P chart, the NP chart focuses on the average number of non-conforming units. As such, choice of the P or NP chart is simply a matter of preference, as each is a scaled version of the other.

P Chart Formulas

Suppose we have k samples, each of size n_i . Let D_i represent the number of nonconforming units in the i^{th} sample.

Formulas for the Points on the Chart

The i^{th} proportion p_i is calculated using

$$p_i = \frac{D_i}{n_i}$$

Estimating the P Chart Center Line

In the P Charts procedure, the center line proportion may be input directly, or it may be estimated from a series of samples. If it is estimated from the samples the formula for the centerline proportion is

$$\bar{p} = \frac{\sum_{i=1}^k D_i}{\sum_{i=1}^k n_i}$$

If all the samples are the same size, this formula reduces to

$$\bar{p} = \frac{\sum_{i=1}^k D_i}{kn} = \frac{\sum_{i=1}^k p_i}{k}$$

P Chart Limits

The lower and upper control limits for the P chart are calculated using the formulas

$$LCL = \bar{p} - m \sqrt{\frac{\bar{p}(1 - \bar{p})}{n_i}}$$

$$UCL = \bar{p} + m \sqrt{\frac{\bar{p}(1 - \bar{p})}{n_i}}$$

where m is a multiplier (usually set to 3) chosen to control the likelihood of false alarms (out-of-control signals when the process is in control).

Runs Tests

The strength of control charts comes from their ability to detect sudden changes in a process that result from the presence of assignable causes. Unfortunately, the P chart is poor at detecting drifts (gradual trends) or small shifts in the process. For example, there might be a positive trend in the last ten samples, but until a proportion goes above the upper control limit, the chart gives no indication that a change has taken place in the process.

Runs tests can be used to check control charts for unnatural patterns that are most likely caused by assignable causes. Runs tests are sometimes called “pattern tests”, “out-of-control” tests, or “zones rules”.

While runs tests may be helpful in identifying patterns or smaller shifts in the proportion, they also increase the likelihood of false positive indications. The rate of false positives is typically measured using the average run length (the average length of a run before a false positive is indicated). When several runs tests are used the average run length of the control chart becomes very short.

In order to perform the runs tests, the control chart is divided into six equal zones (three on each side of the center line). Since the control limit is three sigma limits (three standard deviations of the proportion) in width, each zone is one sigma wide and is labeled A, B, or C, with the C zone being the closest to the center line. There is a lower zone A and an upper zone A. The same is true for B and C. The runs tests look at the pattern in which points fall in these zones.

The runs tests used in this procedure are described below.

Test 1: Any Single Point Beyond Zone A

This runs test simply indicates a single point is beyond one of the two three-sigma limits.

Test 2: Two of Three Successive Points in Zone A or Beyond

This usually indicates a shift in the process average. Note that the two points have to be in the same Zone A, upper or lower. They cannot be on both sides of the center line. The third point can be anywhere.

Test 3: Four of Five Successive Points in Zone B or Beyond

This usually indicates a shift in the process average. Note that the odd point can be anywhere.

Test 4: Eight Successive Points in Zone C or Beyond

All eight points must be on one side of the centerline. This is another indication of a shift in the process average.

Test 5: Fifteen Successive Points Fall in Zone C on Either Side of the Centerline

Although this pattern might make you think that the variation in your process has suddenly decreased, this is usually not the case. It is usually an indication of stratification in the sample. This happens when the samples come from two distinct distributions having different means. Perhaps there are two machines that are set differently. Try to isolate the two processes and check each one separately.

Test 6: Eight of Eight Successive Points Outside of Zone C

This usually indicates a mixture of processes. This can happen when two supposedly identical production lines feed a single production or assembly process. You must separate the processes to find and correct the assignable cause.

There are, of course, many other sets of runs tests that have been developed. You should watch your data for trends, zig-zags, and other nonrandom patterns. Any of these conditions could be an indication of an assignable cause and would warrant further investigation.

Issues in Using Control Charts

There are several additional considerations surrounding the use of control charts that will not be addressed here. Some important questions are presented below without discussion. For a full treatment of these issues you should consider a statistical quality control text such as Ryan (2011) or Montgomery (2013).

Sample Size

How many items should be sampled for each sample? How does the sample size affect my use of control charts? What about unequal sample sizes?

Dealing with Out-of-Control Points

How do you deal with out-of-control points once they have been detected? Should they be included or excluded in the process proportion estimate?

Control Limit Multiplier

Three-sigma limits are very common. When should one consider a value other than three?

Startup Time

How many samples should be used to establish control for my process?

Data Structure

In this procedure, the data are entered in two columns. One column contains the sample size for each sample, the other column contains the number of nonconforming units.

Example Dataset

| N | Nonconf |
|----|---------|
| 50 | 2 |
| 50 | 8 |
| 50 | 6 |
| 50 | 3 |
| 50 | 4 |
| 50 | 2 |
| 50 | 7 |
| 50 | 1 |
| 50 | 9 |
| 50 | 7 |
| 50 | 3 |
| 50 | 5 |
| . | . |
| . | . |
| . | . |

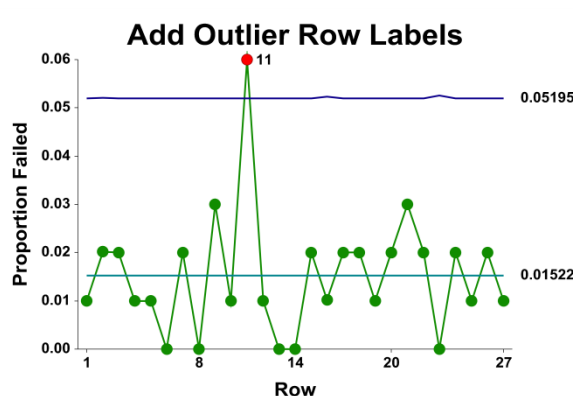
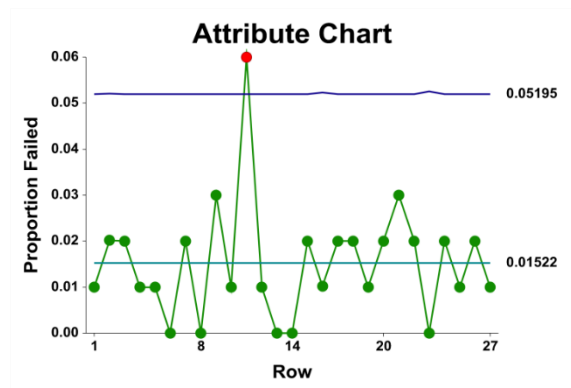
Attribute Chart Format Window Options

This section describes the specific options available on the Attribute Chart Format window, which is displayed when the Attribute Chart Format button is clicked. Common options, such as axes, labels, legends, and titles are documented in the Graphics Components chapter.

Attribute Chart Tab

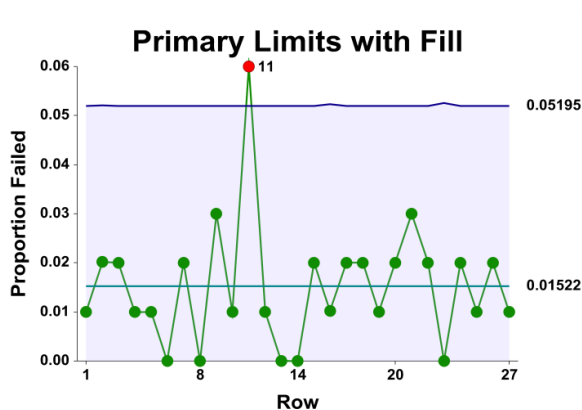
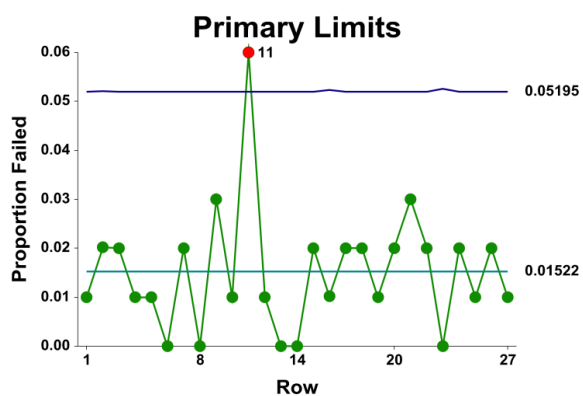
Symbols Section

You can modify the attributes of the symbols using the options in this section.



Lines Section

You can specify the format of the various lines using the options in this section. Note that when shading is desired, the fill will be to the bottom for single lines (such as the mean line), and between the lines for pairs of lines (such as primary limits).



Titles, Legend, Numeric Axis, Group Axis, Grid Lines, and Background Tabs

Details on setting the options in these tabs are given in the Graphics Components chapter. The legend does not show by default but can easily be included by going to the Legend tab and clicking the Show Legend checkbox.

Example 1 – P Chart Analysis (Phase I)

This section presents an example of how to run an initial P Chart analysis to establish control limits. In this example, a specific location of an automotive connection will be examined for a leak. Seventy connections are to be examine each day for 40 days to establish control of the process. The data used are in the Leak dataset.

Setup

To run this example, complete the following steps:

1 Open the Leak example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Leak** and click **OK**.

2 Specify the P Charts procedure options

- Find and open the **P Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Sample Size Variable**Size**
 Number Nonconforming Variable.....**Leak**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Estimation and Limit Section

Estimation and Limit Section for Samples 1 to 40

Number of Samples: 40
 Sample Size Variable: Size
 Number Nonconforming Variable: Leak

| Average Sample Size | Average # Nonconf. | Total Sample Size | Total # Nonconf. | Proportion Nonconf. | Proportion Nonconf. Limits | |
|---------------------|--------------------|-------------------|------------------|---------------------|----------------------------|-------------|
| | | | | | Lower Limit | Upper Limit |
| 70.00 | 7.425000 | 2800 | 297 | 0.1060714 | 0.000000 | 0.216485 |

This section displays the calculation values and limits that are to be used in the P chart. The formulas for each calculation are described in the P chart formula section toward the beginning of this chapter.

P Charts

Average Sample Size

This is the average of all the sample sizes.

Average Number Nonconforming

This is the average number of nonconforming units.

Total Sample Size

This is the sum of all the sample sizes.

Total Number Nonconforming

This is the sum of all the numbers of nonconforming units.

Proportion Nonconforming

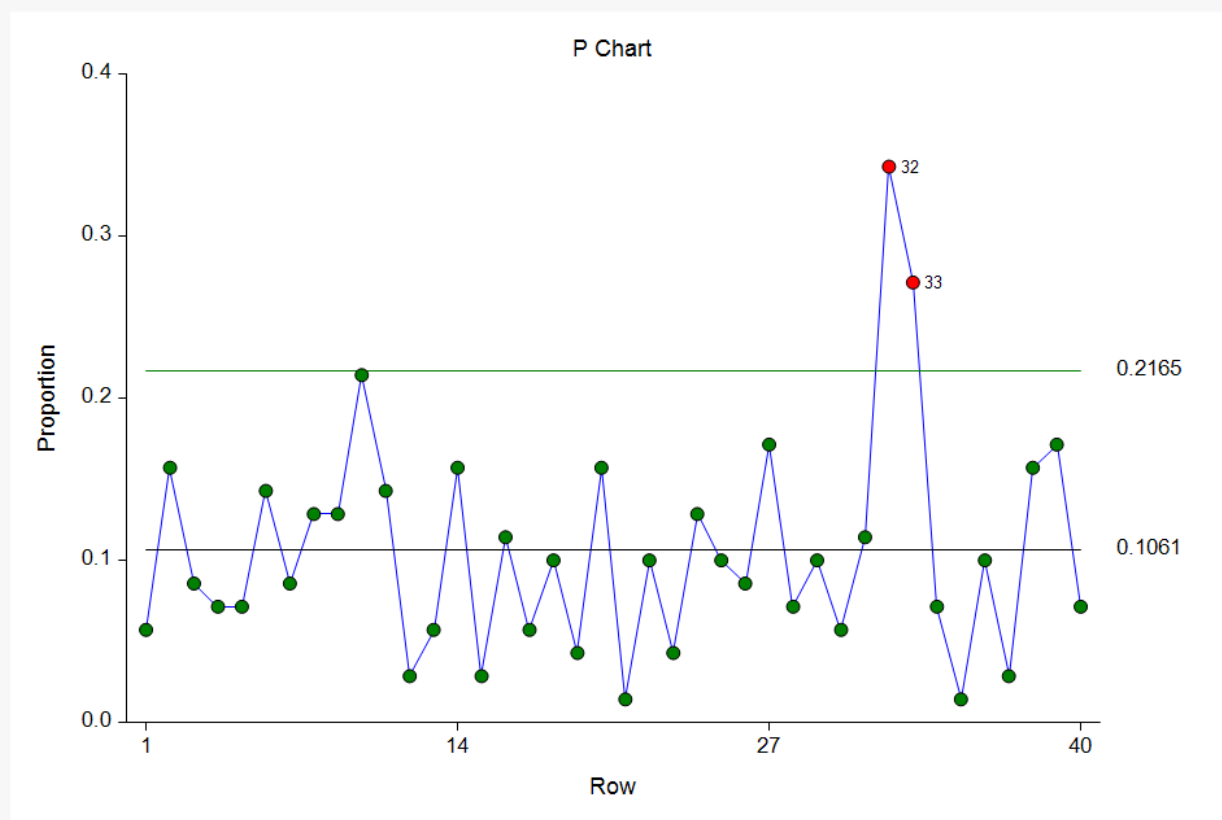
This is the proportion of nonconforming units.

Proportion Nonconforming Limits

These are the lower and upper control limits for the P chart.

P Chart

Chart Section for Samples 1 to 40



P Charts

The P chart shows the sequence of proportions of leaking connections. The chart show that samples 32 and 33 merit further investigation for an assignable cause. After reviewing the records, the investigators found that the 32nd and 33rd samples came during and immediately after the training of a new connection installer.

Out-of-Control List

Out-of-Control List for Samples 1 to 40

| Row | Proportion | Row Label | Reason |
|-----|------------|-----------|-----------------------|
| 32 | 0.342857 | 32 | beyond control limits |
| 33 | 0.271429 | 33 | beyond control limits |
| 34 | 0.071429 | 34 | 2 of 3 in zone A |
| 37 | 0.028571 | 37 | 2 of 3 in zone A |

This report provides a list of the samples that failed one of the runs tests (including points outside the control limits). The report shows that samples 32 and 33 are beyond the control limits. Because the 32nd and 33rd samples came during and immediately after the training of a new connection installer, the investigators determine that the chart limits should be revised to exclude samples 32 and 33. Samples 34 and 37 show runs in the data, but they do seem to signal an important change in the process.

Example 2 – P Chart Revised (Phase I)

This section presents a continuation of the previous example. In this example the limits are revised to exclude the data from samples 32 and 33, due to assignable cause.

Setup

To run this example, complete the following steps:

1 Open the Leak example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Leak** and click **OK**.

2 Specify the P Charts procedure options

- Find and open the **P Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Sample Size Variable **Size**
 Number Nonconforming Variable..... **Leak**
 Specification Method **Keep Rows Variable**
 Use all rows where variable..... **Keep**
 equals **1**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Estimation and Limit Section

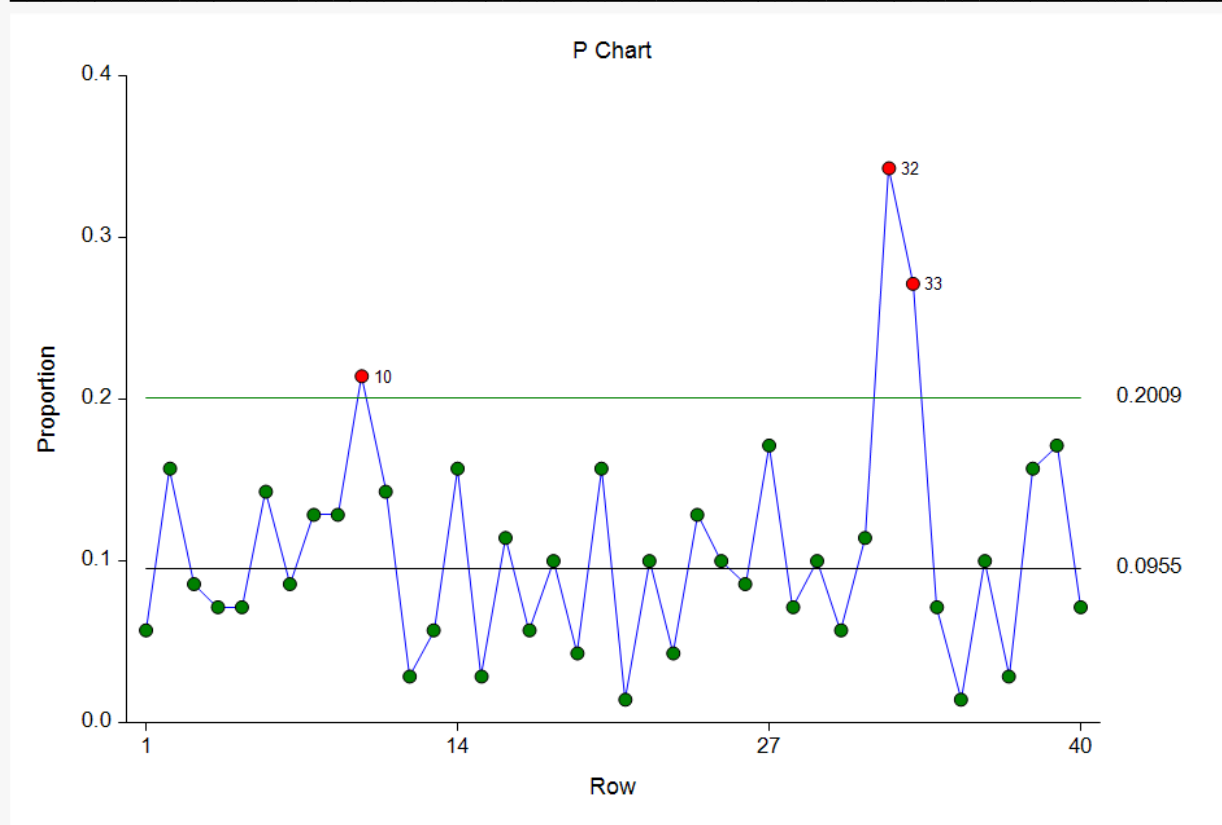
Estimation and Limit Section for Samples 1 to 40 where Keep equals 1

Number of Samples: 38
 Sample Size Variable: Size
 Number Nonconforming Variable: Leak

| Average Sample Size | Average # Nonconf. | Total Sample Size | Total # Nonconf. | Proportion Nonconf. | Proportion Nonconf. Limits | |
|---------------------|--------------------|-------------------|------------------|---------------------|----------------------------|-------------|
| | | | | | Lower Limit | Upper Limit |
| 70.00 | 6.684211 | 2660 | 254 | 0.09548872 | 0.000000 | 0.200868 |

P Charts

Chart Section for Samples 1 to 40



Out-of-Control List for Samples 1 to 40

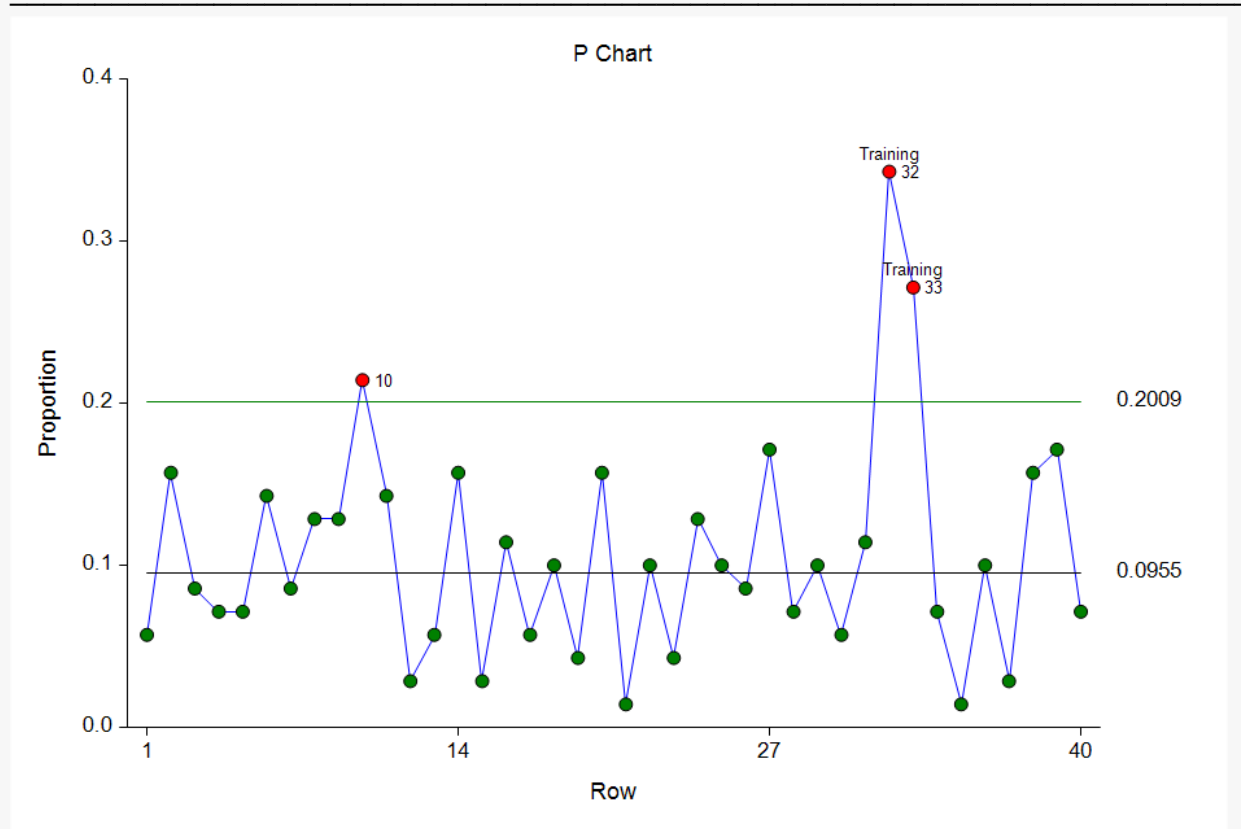
| Row | Proportion | Row Label | Reason |
|-----|------------|-----------|-----------------------|
| 10 | 0.214286 | 10 | beyond control limits |
| 32 | 0.342857 | 32 | beyond control limits |
| 33 | 0.271429 | 33 | beyond control limits |
| 34 | 0.071429 | 34 | 2 of 3 in zone A |

The limits in this report and chart are now based on all samples except samples 32 and 33. The proportions for samples 32 and 33 are included in the chart, but they are not included in the calculations. Because two of the more extreme proportions were removed, the control limits are narrower than those of Example 1. Although sample 10 now gives an out-of-control signal, the investigators did not find an assignable cause and therefore chose to leave sample 10 in the calculations of the center line and limits. Sample 34 gives an out-of-control signal due to samples 32 and 33 being beyond zone A.

P Charts

The P chart can be additionally enhanced by labeling the points that were not included in the calculations. To do this, set the Point Label Variable to Label.

Chart Section for Samples 1 to 40



Example 3 – P Chart Analysis (Phase II)

Continuing with Examples 1 and 2, the investigators obtain samples for an additional 20 days. They wish to use the limits based on the first 40 samples, excluding samples 32 and 33.

Setup

To run this example, complete the following steps:

1 Open the Leak example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Leak** and click **OK**.

2 Specify the P Charts procedure options

- Find and open the **P Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Sample Size Variable **SizeCont**
 Number Nonconforming Variable..... **LeakCont**
 Specification Method **Keep Rows Variable**
 Use all rows where variable..... **KeepCont**
 equals **1**
 Point Label Variable..... **Label**

Report Options (*in the Toolbar*)

Variable Labels **Column Names**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

P Chart Analysis Output

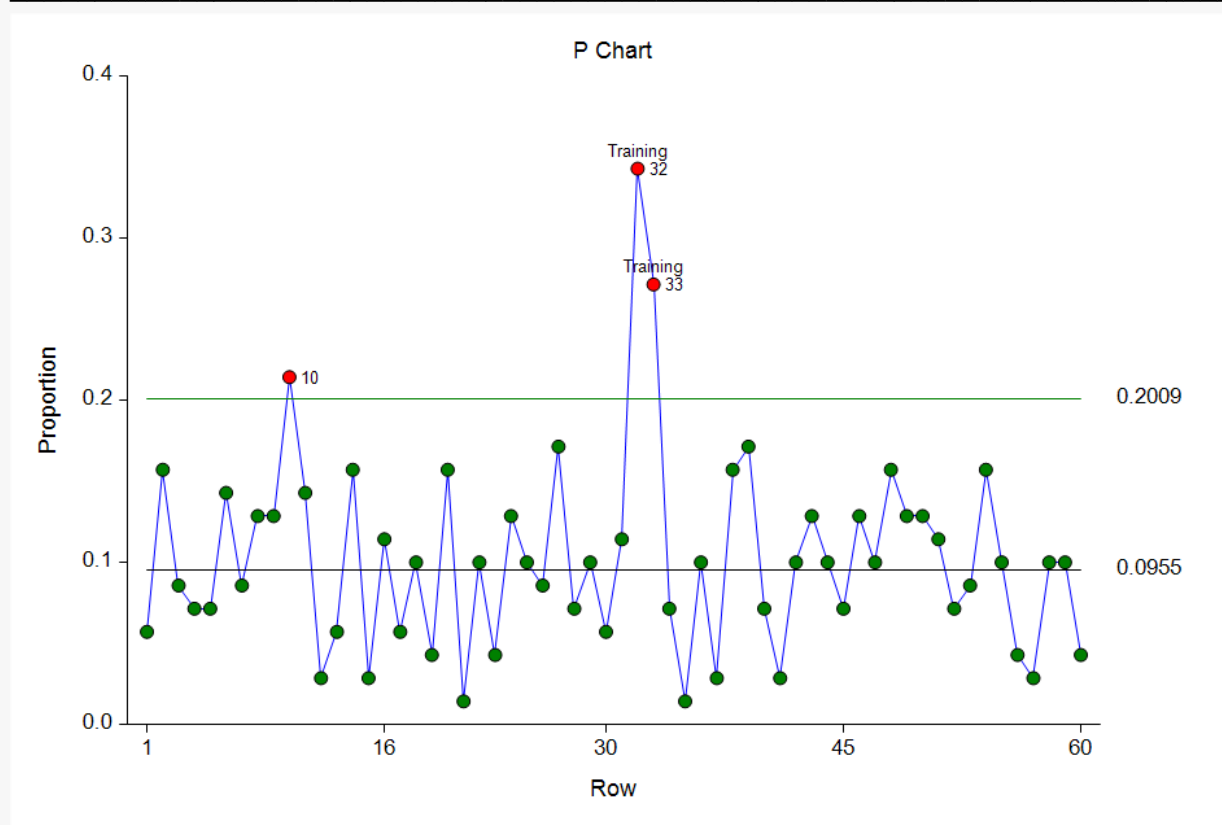
Estimation and Limit Section for Samples 1 to 60 where KeepCont equals 1

Number of Samples: 38
 Sample Size Variable: SizeCont
 Number Nonconforming Variable: LeakCont

| Average Sample Size | Average # Nonconf. | Total Sample Size | Total # Nonconf. | Proportion Nonconf. | Proportion Nonconf. Limits | |
|---------------------|--------------------|-------------------|------------------|---------------------|----------------------------|-------------|
| | | | | | Lower Limit | Upper Limit |
| 70.00 | 6.684211 | 2660 | 254 | 0.09548872 | 0.000000 | 0.200868 |

P Charts

Chart Section for Samples 1 to 60



Out-of-Control List for Samples 1 to 60

| Row | Proportion | Row Label | Reason |
|-----|------------|-----------|-----------------------|
| 10 | 0.214286 | 10 | beyond control limits |
| 32 | 0.342857 | 32 | beyond control limits |
| 33 | 0.271429 | 33 | beyond control limits |
| 34 | 0.071429 | 34 | 2 of 3 in zone A |

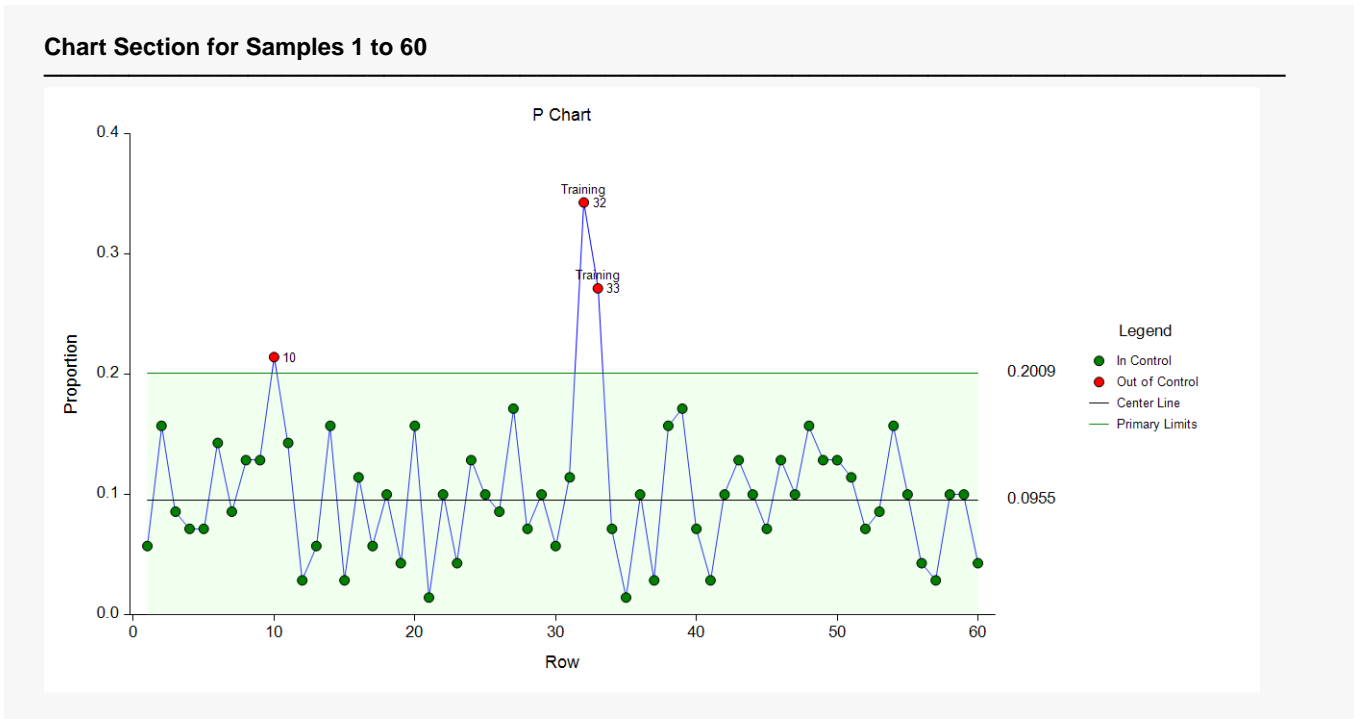
The estimation and limits section shows the same results as Example 2, since the calculations are again based only on the first 40 samples, excluding 32 and 33. The plot shows a stable continuation of the process from samples 41 to 60.

Example 4 – P Chart with Additional Formatting

This example uses the same setup as Example 3, except that a variety of improvements are made in the plot format. These improvements are made by clicking the P Chart format button on the P Chart tab.

The settings for this example are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

P Chart



As shown here, a variety of enhancements can be made to the formatting of the control chart to make the chart as easy to read as possible.

Example 5 – P Chart with Stages

This section presents a continuation of Examples 1, 2, and 3. After the first 60 days, a new connection process is implemented to reduce the number of leaks. The investigators obtained samples of the process for 60 additional days, using the first 30 samples of the additional 60 days (61 to 90) to determine the in-control limits of the updated process. The investigators wish to view the entire progress of the process, beginning with day 1. This example shows the use of stages to monitor a process with a change in center line and limit calculations.

Setup

To run this example, complete the following steps:

1 Open the Leak example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Leak** and click **OK**.

2 Specify the P Charts procedure options

- Find and open the **P Charts** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Sample Size Variable **SizeStage**
 Number Nonconforming Variable..... **LeakStage**
 Number of Stages..... **Multiple Stages**
 Stage Variable **Stage**
 Specification Method **Keep Rows Variable**
 Use all rows where variable..... **KeepStage**
 equals **1**
 Point Label Variable..... **Label**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Estimation and Limit Section

Estimation and Limit Section for Samples 1 to 60 where KeepStage equals 1

Number of Samples: 38
 Sample Size Variable: SizeStage
 Number Nonconforming Variable: LeakStage

| Average Sample Size | Average # Nonconf. | Total Sample Size | Total # Nonconf. | Proportion Nonconf. | Proportion Nonconf. Limits | |
|---------------------|--------------------|-------------------|------------------|---------------------|----------------------------|-------------|
| | | | | | Lower Limit | Upper Limit |
| 70.00 | 6.684211 | 2660 | 254 | 0.09548872 | 0.000000 | 0.200868 |

Estimation and Limit Section for Samples 61 to 120 where KeepStage equals 1

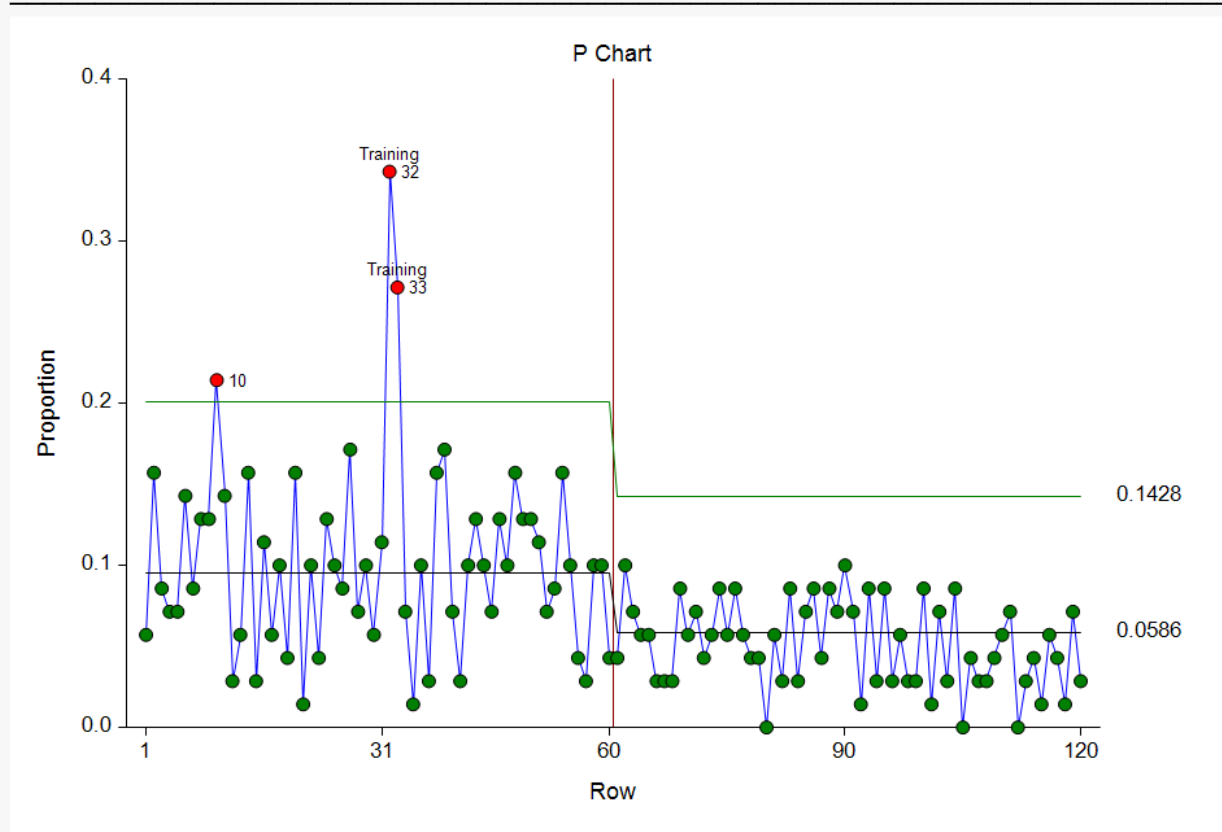
Number of Samples: 30
 Sample Size Variable: SizeStage
 Number Nonconforming Variable: LeakStage

| Average Sample Size | Average # Nonconf. | Total Sample Size | Total # Nonconf. | Proportion Nonconf. | Proportion Nonconf. Limits | |
|---------------------|--------------------|-------------------|------------------|---------------------|----------------------------|-------------|
| | | | | | Lower Limit | Upper Limit |
| 70.00 | 4.100000 | 2100 | 123 | 0.05857143 | 0.000000 | 0.142771 |

This section displays the estimation for each stage separately. The proportion nonconforming is decreased by about 40%.

P Chart

Chart Section for Samples 1 to 120



The P chart shows the substantial decrease in the proportion of leaking connections in the second stage. The center line and limits of the second stage are estimated from samples 61 to 90.

Out-of-Control List

Out-of-Control List for Samples 1 to 60

| Row | Proportion | Row Label | Reason |
|-----|------------|-----------|-----------------------|
| 10 | 0.214286 | 10 | beyond control limits |
| 32 | 0.342857 | 32 | beyond control limits |
| 33 | 0.271429 | 33 | beyond control limits |
| 34 | 0.071429 | 34 | 2 of 3 in zone A |

With no new out-of-control points, the out-of-control list is the same as that of the previous examples. There is not an Out-of-Control List for samples 61 to 120 since there are no out-of-control points in this group.