Chapter 491

Maximum Flow

Introduction

Given a directed network defined by nodes, arcs, and flow capacities, this procedure finds the maximum flow that can occur between a source node and a sink node. An example of this is the flow of oil through a pipeline with several junctions.

There are specialized algorithms that can be used to solve for the maximum flow. Taha (2011) and Hillier and Lieberman (2015) show that the problem can be solved using a linear programming algorithm. This is the approach we have taken in **NCSS**.

Data Structure

This procedure requires a special data format in which the rows represent the arcs (branches) between two locations (nodes). Each arc is defined by two nodes and the positive flow amount. The flow is always from the first node to the second node.

If flow can occur in two directions, a separate row must be added for each flow. This occurs in the example below with nodes N3 and N4.

Note that by definition, you cannot have flow into the source node or out of the sink node. If you try to, an error will occur.

Consider the following material transportation example. Each row represents a pipe (arc) between two junctions (nodes). The junctions are identified by letters and the flow in gallons per unit time. We assume that N1 is the 'source' node and N5 is the 'sink' node.

These data are stored in the dataset *Maximum Flow*.

Maximum Flow Dataset

Node1	Node2	Flow
N1	N2	25
N1	N3	29
N1	N4	11
N2	N3	39
N2	N5	28
N3	N4	12
N3	N5	18
N4	N3	6
N4	N5	22

Linear Programming Formulation of the Maximum Flow Problem

As stated earlier, we use a linear programming algorithm to solve for the maximum. Rather than present all the equations, we show how the above example is translated into a linear programming tableau. The LP tableau for the above data is

		X_{12}	X_{13}	X_{14}	X_{23}	X_{25}	X_{34}	X_{35}	X_{43}	X_{45}	RHS
Maximize	z =	1	1	1							
Nodes											
N2		1			-1	-1					= 0
N3			1		1		-1	-1	1		= 0
N4				1			1		-1	-1	= 0
Capacities (Bounds)		25	29	11	39	28	12	18	6	22	

Example 1 – Maximum Flow through a Pipeline

This section presents an example of how to find the maximal flow between nodes N1 and N5 in the following network. As you use this example, you need to keep in mind that all flows are in only one direction.

Maximum Flow Dataset

Node1	Node2	Flow
N1	N2	25
N1	N3	29
N1	N4	11
N2	N3	39
N2	N5	28
N3	N4	12
N3	N5	18
N4	N3	6
N4	N5	22

Setup

To run this example, complete the following steps:

1   Open the Max Flow example dataset
- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Max Flow** and click **OK**.

2   Specify the Maximum Flow procedure options
- Find and open the **Maximum Flow** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Specifications Tab

Node 1 (Starting) Column **Node1**
Node 2 (Ending) Column **Node2**
Flow Capacity Column....................................**Flow**
Node Values Input Type ....**Enter custom source and sink values**
Source (Starting) Node Value.........................**N1**
Sink (Ending) Node Value **N5**

3   Run the procedure
- Click the **Run** button to perform the calculations and generate the output.

Maximum Flow Solution

Maximum Flow

Arc (Row)	Maximum Flow Through Arc	Flow (Capacity)	Node1 (Start)	Node2 (End)
1	25	25	N1 (Source)	N2
2	29	29	N1 (Source)	N3
3	11	11	N1 (Source)	N4
5	25	28	N2	N5 (Sink)
6	11	12	N3	N4
7	18	18	N3	N5 (Sink)
9	22	22	N4	N5 (Sink)
Total	65			

This report lists the solution by giving the arcs that form the maximal flow. The starting node is identified with the word *Source*. The ending node is identified with the word *Sink*.

The maximum flow through the network is 65.

Possible Network Arcs

Possible Network Arcs

Arc (Row)	Flow (Capacity)	Node1 (Start)	Node2 (End)
1	25*	N1 (Source)	N2
2	29*	N1 (Source)	N3
3	11*	N1 (Source)	N4
4	39	N2	N3
5	28*	N2	N5 (Sink)
6	12*	N3	N4
7	18*	N3	N5 (Sink)
8	6	N4	N3
9	22*	N4	N5 (Sink)

* The starred arcs are part of the maximum flow.

This report lists all the arcs from which the maximal flow was derived. The starting node is identified with the word *Source*. The ending node is identified with the word *Sink*.