Chapter 481

Linear Programming with Bounds

Introduction

Linear programming maximizes (or minimizes) a linear objective function subject to one or more constraints. The technique finds broad use in operations research and is occasionally of use in statistical work.

The mathematical representation of the linear programming (LP) problem is

Maximize (or minimize)

$$z = \boldsymbol{CX}$$

subject to

$$\boldsymbol{AX} \leq \boldsymbol{b}, \boldsymbol{X} \geq \boldsymbol{0}$$

where

$$\boldsymbol{X} = (x_1, x_2, \dots, x_n)'$$

$$\boldsymbol{C} = (c_1, c_2, \dots, c_n)$$

$$\boldsymbol{b} = (b_1, b_2, \dots, b_m)'$$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

The x_i's are the *decision variables* (the unknowns), the first equation is called the *objective function* and the *m* inequalities (and equalities) are called *constraints*. The constraint bounds, the b_i's, are often called *right-hand sides* (RHS).

NCSS solves a particular linear program using a revised dual simplex method available in the *Extreme Optimization* mathematical subroutine package.

Example

We will solve the following problem using **NCSS**:

Maximize

$$z = x_1 + x_2 + 2x_3 - 2x_4$$

subject to

$$x_1 + 2x_3 \leq 700$$

$$2x_2 - 8x_3 \leq 0$$

$$x_2 - 2x_3 + x_4 \geq 1$$

$$x_1 + x_2 + x_3 + x_4 = 10$$

$$0 \leq x_1 \leq 10$$

$$0 \leq x_2 \leq 10$$

$$0 \leq x_3 \leq 10$$

$$0 \leq x_4 \leq 10$$

The solution (see Example 1 below) is x_1 = 9, x_2 = 0.8, x_3 = 0, and x_4 = 0.2 which results in z = 9.4.

Data Structure

This technique requires a special data format which will be discussed under the *Specifications* tab. Here is the way the above example would be entered. It is stored in the dataset *LP 1*.

LP 1 Dataset

Type	Logic	RHS	X1	X2	X3	X4
O			1	1	2	-2
C	<	700	1		2	
C	<	0		2		-8
C	>	1		1	-2	1
C	=	10	1	1	1	1
L			0	0	0	0
U			10	10	10	10

Example 1 – Linear Programming with Bounds

This section presents an example of how to run the data presented in the example given above. The data are contained in the LP 1 database. Here is the specification of the problem.

Maximize

$$z = x_1 + x_2 + 2x_3 - 2x_4$$

subject to

$$x_1 + 2x_3 \leq 700$$

$$2x_2 - 8x_3 \leq 0$$

$$x_2 - 2x_3 + x_4 \geq 1$$

$$x_1 + x_2 + x_3 + x_4 = 10$$

$$0 \leq x_1 \leq 10$$

$$0 \leq x_2 \leq 10$$

$$0 \leq x_3 \leq 10$$

$$0 \leq x_4 \leq 10$$

Setup

To run this example, complete the following steps:

1 Open the LP 1 example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **LP 1** and click **OK**.

2 Specify the Linear Programming with Bounds procedure options

- Find and open the **Linear Programming with Bounds** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Specifications Tab

Type of Optimum .... **Maximum**
Row Type Column .... **Type**
Variables Columns.. **X1-X4**
Labels of Constraints Column......................... **CLabel**
Logic Column.. **Logic**
Constraint Bounds (RHS) Column.................. **RHS**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Objective Function and Solution for Maximum

Objective Function and Solution for Maximum

Variable	Objective Function Coefficient	Value at Maximum
X1	1.0	9.000
X2	1.0	0.800
X3	2.0	0.000
X4	-2.0	0.200
Maximum of Objective Function		9.400

Solution Status: The optimization model is optimal.

This report lists the linear portion of the objective function coefficients and the values of the variables at the maximum (that is, the solution). It also shows the value of the objective function at the solution as well as the status of the algorithm when it terminated.

Constraints

Constraints

Label, Logic	X1	X2	X3	X4	RHS
Con1, ≤	1.0	0.0	2.0	0.0	700.0
Con2, ≤	0.0	2.0	0.0	-8.0	0.0
Con3, ≥	0.0	1.0	-2.0	1.0	1.0
Con4, =	1.0	1.0	1.0	1.0	10.0

This report presents the coefficients of the constraints as they were input.

Values of Constraints at Solution for Maximum and Dual Values

Values of Constraints at Solution for Maximum and Dual Values

Label, Logic	RHS	RHS at Solution	Dual Value
Con1, ≤	700.0	9.000	0.000
Con2, ≤	0.0	0.000	-0.300
Con3, ≥	1.0	1.000	0.600
Con4, =	10.0	10.000	-1.000

This report presents the right-hand side of each constraint along with its value at the optimal values of the variables.