Chapter 469

Decomposition Forecasting

Introduction

Classical time series decomposition separates a time series into five components: mean, long-range trend, seasonality, cycle, and randomness. The decomposition model is

Value = (Mean) x (Trend) x (Seasonality) x (Cycle) x (Random)

Note that this model is multiplicative rather than additive. Although additive models are more popular in other areas of statistics, forecasters have found that the multiplicative model fits a wider range of forecasting situations.

Decomposition is popular among forecasters because it is easy to understand (and explain to others). While complex ARIMA models are often popular among statisticians, they are not as well accepted among forecasting practitioners. For seasonal (monthly, weekly, or quarterly) data, decomposition methods are often as accurate as the ARIMA methods, and they provide additional information about the trend and cycle which may not be available in ARIMA methods.

Decomposition has one disadvantage: the cycle component must be input by the forecaster since it is not estimated by the algorithm. You can get around this by ignoring the cycle, or by assuming a constant value. Some forecasters consider this a strength because it allows the forecaster to enter information about the current business cycle into the forecast.

Decomposition Method

The basic decomposition method consists of estimating the five components of the model

$$X_t = UT_tC_tS_tR_t$$

where

X_t denotes the series or, optionally, log of series.

U denotes the mean of the series.

T_t denotes the linear trend.

C_t denotes cycle.

S_t denotes season.

R_t denotes random error.

t denotes the time period.

We will now take you through the steps used by the program to perform a decomposition of a time series. Most of this information is from Makridakis (1978), chapter 15.

Step 1 – Remove the Mean

The first step is to remove the mean by dividing each individual value by the series mean. This creates a new series that has values near one. This step is represented symbolically as

$$Y_t = X_t \,/\, U$$

If the absolute value of the mean of the series is less than 0.0000001, no division takes place.

Step 2 – Calculate a Moving Average

The next step calculates an L-step moving average centered at the time period, *t*, where *L* is the length of the seasonality (e.g., *L* would be 12 for a monthly series or 4 for quarterly series). Since the moving average gives the mean of a year's data, the seasonality factor is removed. Usually, the averaging removes the randomness component as well. Symbolically, this step is represented as

$$M_t = \sum Y_t$$

where for odd *L*, the summation runs from *t - [L/2]* to *t + [L/2].* The symbols *[x]* mean take the integer part of *x*. Hence [6.43] = 6 and [11/2] = 5. Notice that this summation range centers the moving average at *t*.

For even *L*, the values usually found in practice (2, 4 and 12), it is a little more difficult to center the moving average on the time period *t*. For example, the average of the first 12 terms of a series would be centered at 6.5 rather than 6. To center the average right on 7, we must compute the moving average centered at 6.5 and at 7.5 and then average these. The resulting double moving average is centered at the desired value of 7.

Another complexity that must be dealt with is what to do at the ends of the series. Because the average is centered, the first and last *L/2* averages cannot be computed (because of the lack of data). Many different end-effect techniques have been proposed.

Our end-effect strategy can best be explained by considering an example. Suppose we have a monthly series that runs from January of 1980 to December of 1988. To compute the moving average centered at January, 1980, we will need estimated data back through July, 1979. The estimate of July 1979 is obtained by subtracting the difference of July, 1980 and July, 1981 from July, 1980.

At the other end of the series we will need estimated values through June, 1989. To compute the estimated value for June, 1989, we add the difference of June, 1987 and June, 1988 to June, 1988.

This method of estimating end-effects preserves local trends in the series. However, it is especially sensitive to outliers. You should remember that strange patterns in the last *L/2* time periods may be from the end-effect calculation and not from a pattern in the series itself.

Step 3 – Calculate the Trend

The next step is to calculate and remove the trend component of the series. This calculation is made on the moving averages, M_t, rather than on the Y_t series. A least squares fit is made of the of the model

$$M_t = a + bt + e_t$$

where

a is the intercept.

b is the slope.

e_t is the residual or lack-of-linear-fit.

The linear portion of the above model is used to define the trend. That is, we use

$$T_t = a + bt$$

Note that because of the problems of end-effects, the first and last *L/2* terms are omitted in the trend calculation.

Step 4 – Calculate the Cycle

The cycle term is found by dividing the moving average by the computed trend. Symbolically, this is

$$C_t = \frac{M_t}{T_t}$$

Step 5 – Calculate the Seasonality

The seasonality is computed by dividing the *Y* series by the moving averages. Symbolically, this is

$$K_t = \frac{Y_t}{M_t}$$

Note that the *K* series is composed of both the seasonality and the randomness. To calculate the seasonal component for each season, we simple average all like seasons. That is, the average of all Januarys is computed, the average of all Februarys gives the seasonal value for February, and so on. Mathematically, this is stated as

$$S_g = \sum K_t$$

where the summation is over all *t* in which the season is *g*.

Step 6 – Calculate the Randomness

The final step is to calculate the randomness component. This is accomplished by dividing the K series by S_i where the values of $S_1, S_2, \ldots, S_g$ are repeated as needed. This is represented mathematically as follows

$$R_t = \frac{K_t}{S_t}$$

Creating Forecasts

Once the series decomposition is complete, forecasts may be generated fairly easily. The trend component is calculated using

$$T_t = a + bt$$

the seasonal factor is read from

$$S_g = \sum K_t$$

the cycle factor is input by hand, and the random factor is assumed to be one. If the series was transformed using the log transformation, the forecasts are transformed back using the appropriate inverse function.

Assumptions and Limitations

These algorithms are useful for forecasting seasonal time series with (local or global) trend.

Data Structure

The data are entered in a single column.

This section describes the options available in this procedure.

Missing Values

When missing values are found in the series, they are either replaced or omitted. The replacement value is the average of the nearest observation in the future and in the past or the nearest non-missing value in the past.

If you do not feel that this is a valid estimate of the missing value, you should manually enter a more reasonable estimate before using the algorithm. These missing value replacement methods are particularly poor for seasonal data. We recommend that you replace missing values manually before using the algorithm.

Example 1 – Decompositions Forecasting

This section presents an example of how to generate forecasts of a series using the time series decomposition forecasting method. The data in the Sales dataset will be used. We will forecast the values of the Sales variable for the next twelve months.

Setup

To run this example, complete the following steps:

1 Open the Sales example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Sales** and click **OK**.

2 Specify the Decomposition Forecasting procedure options

- Find and open the **Decomposition Forecasting** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Time Series Variable .... **Sales**
First Year **1970**

Reports Tab

Forecast Report .... **Data and Forecasts**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Forecast Summary Section

Forecast Summary Section

Forecast:	(Mean) x (Trend) x (Cycle) x (Season)
Variable:	Sales
Number of Rows:	144
Missing Values:	None
Mean:	174.2847
Pseudo R-Squared:	0.9872795
Forecast Std. Error:	3.211923
Trend Equation:	Trend = (0.762387) + (0.003224) * (Time Season Number)
Number of Seasons:	12
First Year:	1970
First Season:	1

Seasonal Component Ratios

No.	Ratio	No.	Ratio	No.	Ratio	No.	Ratio
1	0.901933	2	0.855207	3	0.971591	4	0.998289
5	1.030011	6	1.028496	7	0.997262	8	1.005337
9	0.975528	10	1.024248	11	1.007887	12	1.205982

This report summarizes the forecast equation.

Variable

The name of the variable for which the forecasts are generated.

Number of Rows

The number of rows that were in the series. This is provided to allow you to double-check that the correct series was used.

Missing Values

If missing values were found, this option lists the method used to estimate them.

Mean

The mean of the variable across all time periods.

Pseudo R-Squared

This value generates a statistic that acts like the R-Squared value in multiple regression. A value near zero indicates a poorly fitting model, while a value near one indicates a well-fitting model. The statistic is calculated as follows:

$$R^2 = 100\left(1 - \frac{SSE}{SST}\right)$$

where *SSE* is the sum of square residuals and *SST* is the total sum of squares after correcting for the mean.

Forecast Std. Error

The estimated standard deviation of the forecast errors (the difference between the actual and predicted). This value is calculated by squaring and summing all of the forecast errors, dividing by the number of observations, and taking the square root.

Trend Equation

The equation used to predict the trend. The equation is

$$\text{Trend} = a + bt$$

where

a is the intercept.

b is the slope.

t is the time period.

Note that the trend value obtained from this equation will be a ratio type value that will be multiplied by the mean to obtain the actual forecast.

Number of Seasons

The number of rows per year. For example, monthly data would have a value of 12.

First Year

The value of the first year of the series.

First Season

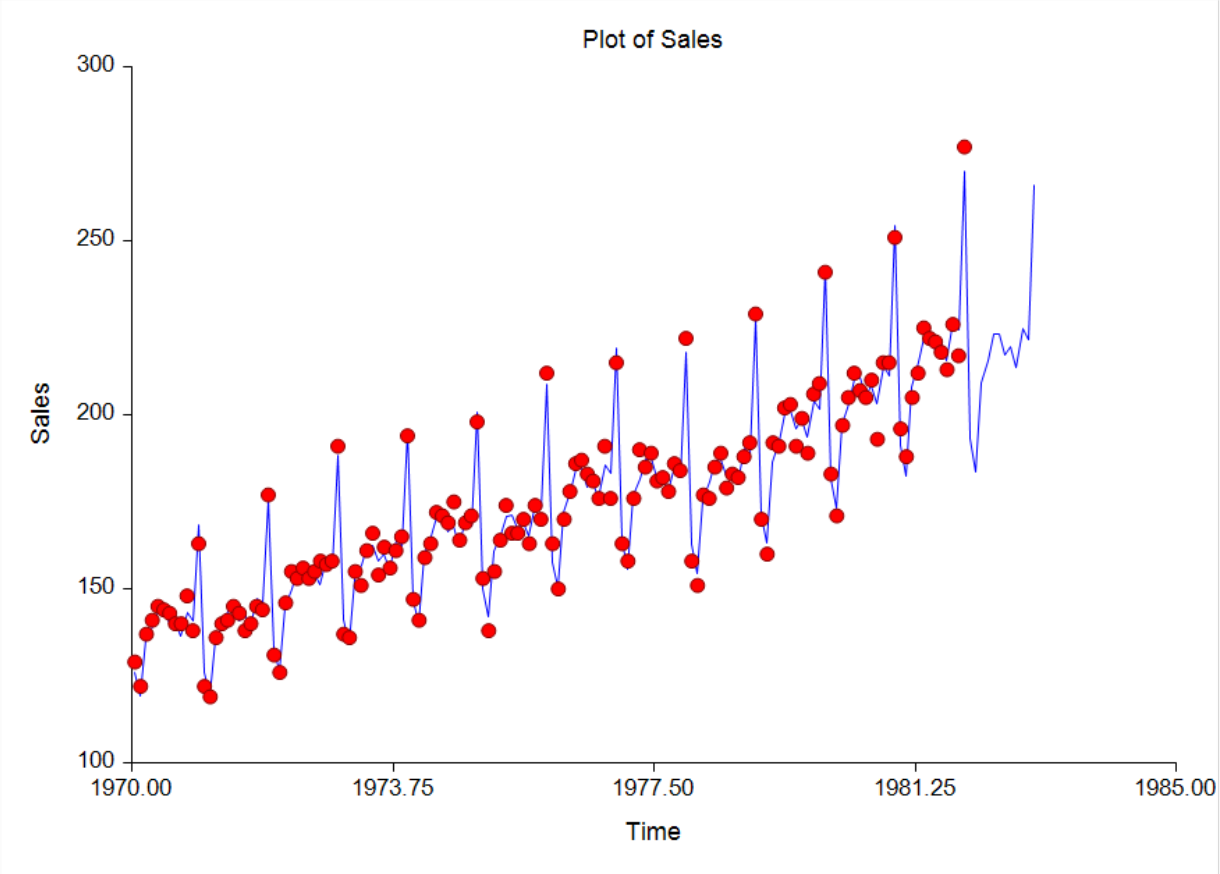
The value of the season of the first observation.

Season Component Ratios

The ratios used to adjust for each season (month or quarter). For example, the last ratio in this example is 1.205982. This indicates that the December correction factor is a 20.5982% increase in the forecast.

Forecast and Data Plot

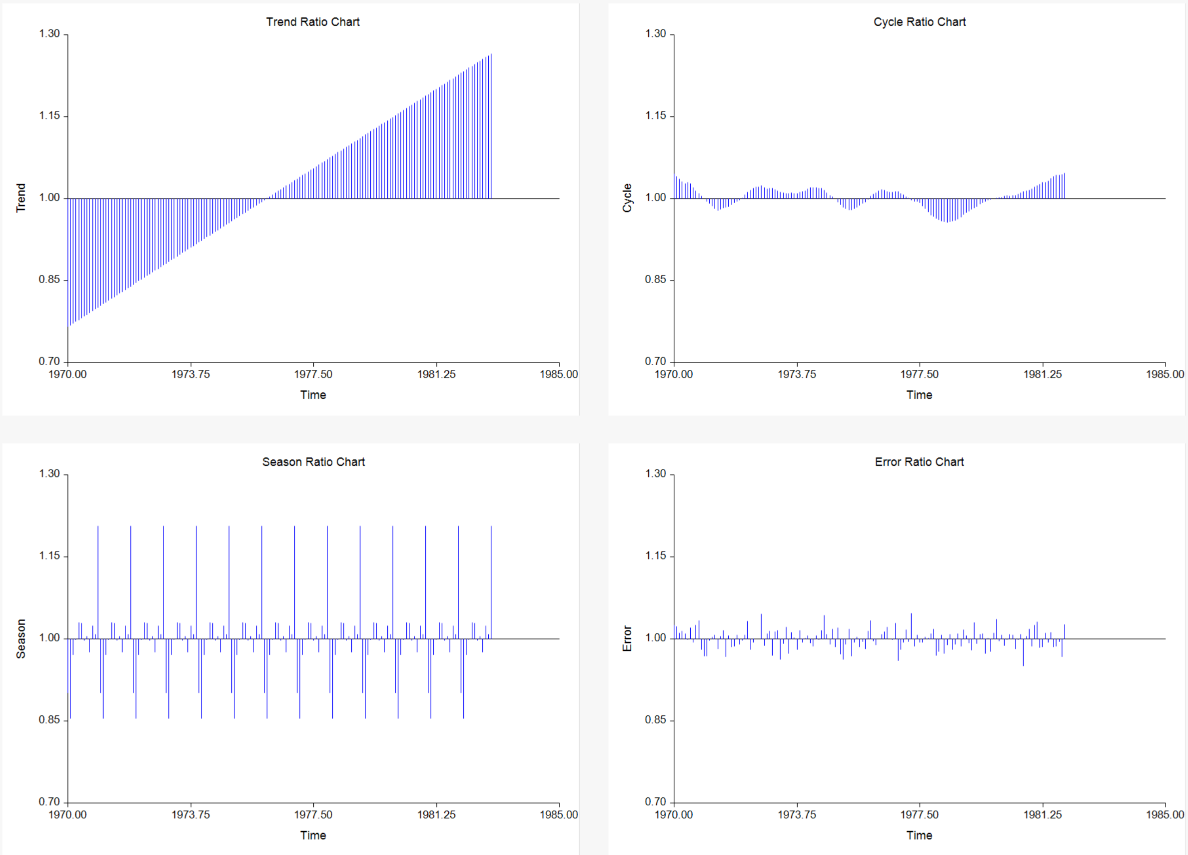
Forecast and Data Plot



The data plot lets you analyze how closely the forecasts track the data. The plot also shows the forecasts at the end of the data series.

Ratio Plots

Decomposition Ratio Plots



Ratio Plots

These plots let you see the various components of the forecast. Each of these plots is centered at one since this is the value that will leave the forecast unchanged. By studying these plots, you can see which factors influence the forecasts the most.

Forecasts Section

Forecasts Section

Row No.	Year Season	Forecast Sales	Actual Sales	Residual	Trend Factor	Cycle Factor	Season Factor	Error Factor
1	1970 1	125.8949	129	3.105124	0.7656	1.0461	0.9019	1.0247
2	1970 2	119.3013	122	2.698687	0.7688	1.0411	0.8552	1.0226
3	1970 3	135.456	137	1.544002	0.7721	1.0361	0.9716	1.0114
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
144	1981 12	269.8886	277	7.111356	1.2266	1.0468	1.2060	1.0263
145	1982 1	193.3271			1.2299	1.0000	0.9019	1.0000
146	1982 2	183.7919			1.2331	1.0000	0.8552	1.0000
147	1982 3	209.3499			1.2363	1.0000	0.9716	1.0000
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

This section shows the values of the forecasts, the dates, the actual values, the residuals, and the forecast ratios.

Note that the forecasted cycle ratios are all equal to one. This is because we did not supply cycle values to be used. If we had, they would have shown up here.