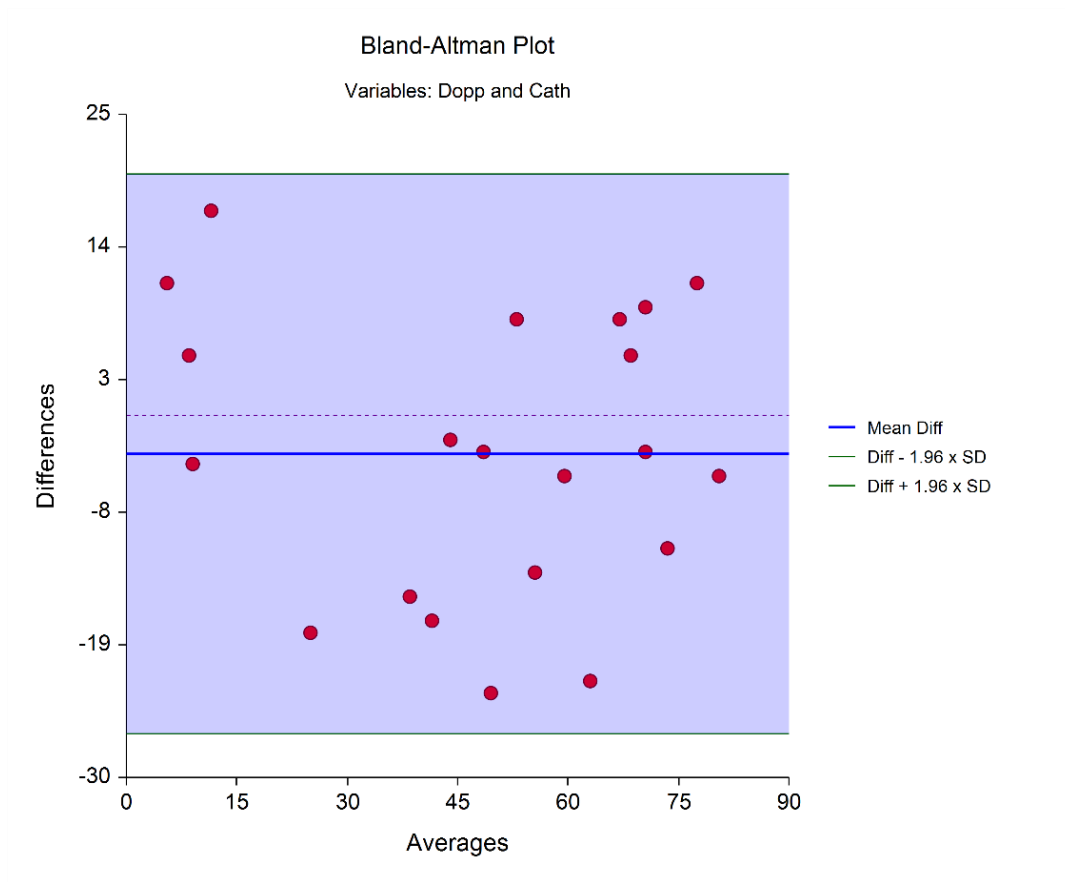


Chapter 204

Bland-Altman Plot and Analysis

Introduction

The Bland-Altman (mean-difference or limits of agreement) plot and analysis is used to compare two measurements of the same variable. That is, it is a method comparison technique. For example, an expensive measurement system might be compared with a less expensive one or an intrusive measurement system might be compared to one that is less intrusive. The technique is documented in a series of papers by J. Martin Bland and Douglas G. Altman (1983, 1986, and 1999).



Repeatability

An important part of method comparison is to understand how repeatable the measurement system is. This can only be understood by sampling each subject multiple times on the same method. This provides for the analysis of designs that include replicates.

Technical Details for Three Designs

There are three study designs that can be analyzed by this procedure. Each type has different input and different technical details, and the output is not identical. The technical details of each design will be presented here.

Design 1: Exactly one data-pair per subject

This is the design that has been used for many years. In this design, each of the two measurement-methods is measured once on each subject at nearly the same point in time. A big drawback of this design is that no repeatability parameter can be computed.

Data Structure

For this design, the data are entered in two columns.

X1	X2
57	53
63	69
66	63
74	76
77	75
77	79
78	77
79	77
80	81
81	82
81	81
82	83
82	81
83	85
84	83
85	74
86	88
86	87
86	82
87	88

Suppose you want to evaluate the agreement between a continuous random variable X_1 and a second random variable X_2 which each measure the same variable, such as blood pressure. Assume that n paired observations (X_{1k}, X_{2k}) , $k = 1, 2, \dots, n$ are available. The Bland-Altman plot (1983) is formed by plotting the differences $X_1 - X_2$ on the vertical axis versus the averages $(X_1 + X_2)/2$ on the horizontal axis. A horizontal line representing the *bias* is drawn at \bar{d} . Additional horizontal lines, known as *limits of agreement*, are added to the plot at $\bar{d} - 1.96 S_d$ and $\bar{d} + 1.96 S_d$. The d 's are the differences formed as $d = X_1 - X_2$.

Sometimes the '1.96' is replaced with '2' or with another value. Of course, 1.96 represents the z-value used to form 95% limits for a unit-normal random variable.

Bland-Altman Plot and Analysis

Bias

The bias between the two tests is measured by the mean of the differences calculated in the usual fashion as

$$\bar{d} = \frac{1}{n} \sum_{k=1}^n d_k$$

Limits of Agreement

Limits of agreement between the two tests are defined by a 95% prediction interval of a particular value of the difference which are computed as follows

$$\bar{d} \pm 1.96S_d$$

where

$$S_d = \sqrt{\frac{1}{n-1} \sum_{k=1}^n (d_k - \bar{d})^2}$$

Bland and Altman (1999) provide the following variances and confidence intervals for the bias and the limits of agreement, assuming that the differences are normally distributed.

Variances

$$\widehat{\text{var}}(\bar{d}) = \frac{S_d^2}{n}$$

$$\widehat{\text{var}}(\bar{d} \pm 1.96S_d) = \left(\frac{1}{n} + \frac{1.96^2}{2(n-1)} \right) S_d^2$$

Confidence Intervals

Hence, 95% confidence intervals for the mean difference (the bias) are

$$\bar{d} \pm t_{1-\alpha/2, n-1} \sqrt{\widehat{\text{var}}(\bar{d})}$$

The 95% confidence intervals for the limits of agreement are

$$(\bar{d} - 1.96 S_d) - t_{1-\alpha/2, n-1} \sqrt{\widehat{\text{var}}(\bar{d} \pm 1.96 S_d)}$$

and

$$(\bar{d} + 1.96 S_d) + t_{1-\alpha/2, n-1} \sqrt{\widehat{\text{var}}(\bar{d} \pm 1.96 S_d)}$$

These confidence intervals provide a measure of the precision of these values that aids in the interpretation of the plot.

Design 2: Multiple replicates for each method, no pairing

In this Bland-Altman design, each subject is measured several times (usually in immediate succession) on one method and then measured several times on the other method. There is no natural pairing of the measures. In fact, the number of replicates does not have to be the same for each method. It is assumed that the overall response mean stays constant throughout the data gathering period.

Data Structure

For this design, all measurements for a specific subject are entered on one row. That is, each row represents a different subject. In the example below, there were two measurements of method X followed by three measurements of method Y.

X1	X2	Y1	Y2	Y3
57	53	56	55	51
63	69	62	64	66
66	63	67	64	65
74	76	76	77	77
77	75	74	76	73
77	79	75	76	78
78	77	73	78	74
79	77	78	79	76
80	81	82	84	83
81	82	80	84	85
81	85	84	83	80

Limits of Agreement Calculation

Suppose you want to evaluate the agreement between a continuous random variable X and a second random variable Y which each measure the same underlying variable, such as blood pressure. Assume that n subjects are available. Variables X and Y are measured repeatedly on each subject. Variable X is measured m_x times and variable Y is measured m_y times. These measurements are not made in X, Y pairs. In fact, all measurements of one method are made in rapid succession followed by all measurements of the other method. The order in which the methods are measured is random for each subject.

The following details come from Zou (2013). Data values x_{ij} and y_{ij} are assumed to follow one-way random effects models

$$x_{ij} = \mu_x + a_{xi} + e_{xij}$$

and

$$y_{ij} = \mu_y + a_{yi} + e_{yij}$$

It is assumed that the quantities a_{xi} , a_{yi} , e_{xij} , and e_{yij} are normal variates with means 0 and variances σ_{xb}^2 , σ_{yb}^2 , σ_{xw}^2 , and σ_{yw}^2 , respectively.

Now, make the following computations.

Bland-Altman Plot and Analysis

Step 1. Compute individual subject means and variances.

$$\bar{x}_i = \frac{1}{m_{xi}} \sum_{j=1}^{m_{xi}} x_{ij}, \quad \bar{y}_i = \frac{1}{m_{yi}} \sum_{j=1}^{m_{yi}} y_{ij}, \quad \bar{d}_i = \bar{x}_i - \bar{y}_i, \quad \bar{d} = \sum_{i=1}^n \frac{d_i}{n}$$

$$s_{xi}^2 = \sum_{j=1}^{m_{xi}} \frac{(x_{ij} - \bar{x}_i)^2}{m_{xi} - 1}, \quad s_{yi}^2 = \sum_{j=1}^{m_{yi}} \frac{(y_{ij} - \bar{y}_i)^2}{m_{yi} - 1}, \quad s_{\bar{d}}^2 = \sum_{i=1}^n \frac{(\bar{d}_i - \bar{d})^2}{n - 1}$$

Step 2. Compute pooled estimates of the within subject random errors.

$$\bar{x}_i = \frac{1}{m_{xi}} \sum_{j=1}^{m_{xi}} x_{ij}, \quad \bar{y}_i = \frac{1}{m_{yi}} \sum_{j=1}^{m_{yi}} y_{ij}$$

$$s_{xw}^2 = \sum_{i=1}^n \frac{m_{xi} - 1}{N_x - 1} s_{xi}^2, \quad s_{yw}^2 = \sum_{i=1}^n \frac{m_{yi} - 1}{N_y - 1} s_{yi}^2$$

where

$$N_x = \sum_{i=1}^n m_{xi}, \quad N_y = \sum_{i=1}^n m_{yi}$$

Step 3. Compute the harmonic means of the replicate counts.

$$m_{xh} = \frac{n}{\sum_{i=1}^n \frac{1}{m_{xi}}}, \quad m_{yh} = \frac{n}{\sum_{i=1}^n \frac{1}{m_{yi}}}$$

Step 4. Compute the standard deviation of a difference.

$$s_{\bar{d}}^2 = s_{\bar{d}}^2 + \left(1 - \frac{1}{m_{xh}}\right) s_{xw}^2 + \left(1 - \frac{1}{m_{yh}}\right) s_{yw}^2$$

Step 5. Finally, compute the limits of agreement.

$$LoA_{lower} = \bar{d} - z_{\beta/2} s_{\bar{d}}$$

$$LoA_{upper} = \bar{d} + z_{\beta/2} s_{\bar{d}}$$

where $z_{\beta/2}$ is the value from the standard normal distribution that puts $\beta/2$ in each tail. Usually, $z_{\beta/2}$ is set to 1.96 or rounded-off to 2.

Confidence interval estimation for LoA based on the delta method

(1 - α)% confidence intervals can be calculated for the lower and upper LoA values using a variance based on the delta method. This variance is computed using

$$\widehat{Var}(LoA_{lower}) = \widehat{Var}(LoA_{upper}) = \frac{s_d^2}{n} + \frac{z_{\beta/2}^2}{2s_d^2} \left[\frac{(s_d^2)^2}{n-1} + \left(1 - \frac{1}{m_{xh}}\right)^2 \frac{(s_{xw}^2)^2}{N_x - n} + \left(1 - \frac{1}{m_{yh}}\right)^2 \frac{(s_{yw}^2)^2}{N_y - n} \right]$$

Confidence Intervals

Hence, (1 - α)% confidence intervals for the two LoA are

$$LoA_{lower} \pm |z_{\alpha/2}| \sqrt{\widehat{Var}(LoA_{lower})}$$

and

$$LoA_{upper} \pm |z_{\alpha/2}| \sqrt{\widehat{Var}(LoA_{upper})}$$

Confidence interval estimation for LoA based on the MOVER method

The above confidence intervals are symmetric and may not be accurate for typical sample sizes. Zou provides the following adjusted confidence interval which simulation studies show to be more accurate in small to moderate sample sizes.

Step 1. Compute l and u as follows.

$$l = s_d^2 - S_1, \quad u = s_d^2 + S_1$$

where

$$S_1 = \sqrt{\left[s_d^2 \left(1 - \frac{n-1}{\chi_{1-\frac{\alpha}{2}, n-1}^2}\right) \right]^2 + \left[\left(1 - \frac{1}{m_{xh}}\right) \left(1 - \frac{N_x - n}{\chi_{1-\frac{\alpha}{2}, N_x - n}^2}\right) s_{xw}^2 \right]^2 + \left[\left(1 - \frac{1}{m_{yh}}\right) \left(1 - \frac{N_y - n}{\chi_{1-\frac{\alpha}{2}, N_y - n}^2}\right) s_{yw}^2 \right]^2}$$

Step 2. Compute LME and RME as follows.

$$LME = \sqrt{\frac{z_{\alpha/2}^2 s_d^2}{n} + z_{\beta/2}^2 \left(\sqrt{u} - \sqrt{s_d^2}\right)^2}$$

$$RME = \sqrt{\frac{z_{\alpha/2}^2 s_d^2}{n} + z_{\beta/2}^2 \left(\sqrt{l} - \sqrt{s_d^2}\right)^2}$$

Bland-Altman Plot and Analysis

Step 3. Compute the MOVER confidence intervals as follows.

The $(1 - \alpha)\%$ MOVER confidence interval for the lower limit of agreement is

$$LoA_{lower} - LME, LoA_{lower} + RME$$

The $(1 - \alpha)\%$ MOVER confidence interval for the upper limit of agreement is

$$LoA_{upper} - RME, LoA_{upper} + LME$$

Design 3: Multiple replicates for each method obtained as pairs

In this Bland-Altman design, each subject is measured by each method several times. At each measurement point, a value for each method is obtained in rapid succession. These are referred to as measurement pairs. It is assumed that the overall response mean varies during the data gathering period.

Data Structure

For this design, each measurement pair is reported on a single row. Three columns of data are required: one with the subject value, one with the measurement of method 1, and one with the measurement of method 2.

Subject	X	Y
1	53	56
1	69	62
2	63	67
2	76	76
2	75	74
3	79	75
3	77	73
4	77	78
4	81	82
4	82	80
4	85	84

Limits of Agreement Calculation

Suppose you want to evaluate the agreement between a continuous random variable X and a second random variable Y which each measure the same underlying variable, such as blood pressure. Assume that n subjects are available. Variables X and Y are measured together repeatedly on each subject m times. Data values $d_{ij} = x_{ij} - y_{ij}$ are assumed to follow a one-way random-effects model

$$d_{ij} = \mu_d + a_i + e_{ij}$$

It is assumed that the quantities a_i and e_{ij} are normal variates with means 0 and variances σ_b^2 and σ_w^2 , respectively.

Bland-Altman Plot and Analysis

In this case, there are two ways to compute the mean difference, neither of which is always better than the other. These two calculation methods may be defined as using the mean of the means and using the mean of the individual differences. These methods require different calculations and will be defined separately.

Limits of Agreement Calculation – Subject Differences.

The following details come from Zou (2013). Now, make the following computations.

Step 1. Compute various means and variances.

$$\bar{d}_i = \sum_{j=1}^{m_i} \frac{d_{ij}}{m_i}, \quad \bar{d} = \sum_{i=1}^n \frac{d_i}{n}, \quad s_i^2 = \sum_{j=1}^{m_i} \frac{(d_{ij} - \bar{d}_i)^2}{m_i - 1}$$

Step 2. Compute pooled estimate of the within subject random error.

$$s_{dw}^2 = \sum_{i=1}^n \frac{m_i - 1}{N - n} s_i^2$$

where

$$N = \sum_{i=1}^n m_i$$

Step 3. Compute pooled estimate of the between subject random error.

$$s_{\bar{d}}^2 = \sum_{i=1}^n \frac{(\bar{d}_i - \bar{d})^2}{n - 1}$$

Step 4. Compute the harmonic mean of the replicate counts.

$$m_h = \frac{n}{\sum_{i=1}^n \frac{1}{m_i}}$$

Step 4. Compute the standard deviation of a difference.

$$s_d^2 = s_{\bar{d}}^2 + \left(1 - \frac{1}{m_h}\right) s_{dw}^2$$

Bland-Altman Plot and Analysis

Step 5. Finally, compute the limits of agreement.

$$LoA_{lower} = \bar{d} - z_{\beta/2} s_d$$

$$LoA_{upper} = \bar{d} + z_{\beta/2} s_d$$

where $z_{\beta/2}$ is the value from the standard normal distribution that puts $\beta/2$ in each tail. Usually, $z_{\beta/2}$ is set to 1.96 or rounded-off to 2.

Confidence interval estimation for LoA based on the delta method

$(1 - \alpha)\%$ confidence intervals can be calculated for the lower and upper LoA values using a variance based on the delta method. This variance is computed using

$$\widehat{Var}(LoA_{lower}) = \widehat{Var}(LoA_{upper}) = \frac{s_d^2}{n} + \frac{z_{\beta/2}^2}{2s_d^2} \left[\frac{(s_d^2)^2}{n-1} + \left(1 - \frac{1}{m_h}\right)^2 \frac{(s_{dw}^2)^2}{N-n} \right]$$

Confidence Intervals

Hence, $(1 - \alpha)\%$ confidence intervals for the two LoA are

$$LoA_{lower} \pm |z_{\alpha/2}| \sqrt{\widehat{Var}(LoA_{lower})}$$

and

$$LoA_{upper} \pm |z_{\alpha/2}| \sqrt{\widehat{Var}(LoA_{upper})}$$

Confidence interval estimation for LoA based on the MOVER method

The above confidence intervals are symmetric and may not be accurate for typical sample sizes. Zou provides the following adjusted confidence interval which simulation studies show to be more accurate in small to moderate sample sizes.

Step 1. Compute l and u as follows.

$$l = s_d^2 - \sqrt{\left[s_d^2 \left(1 - \frac{n-1}{\chi_{1-\frac{\alpha}{2}, n-1}^2} \right) \right]^2 + \left[\left(1 - \frac{1}{m_h} \right) \left(1 - \frac{N-n}{\chi_{1-\alpha/2, N-n}^2} \right) s_{dw}^2 \right]^2}$$

$$u = s_d^2 + \sqrt{\left[s_d^2 \left(1 - \frac{n-1}{\chi_{1-\frac{\alpha}{2}, n-1}^2} \right) \right]^2 + \left[\left(1 - \frac{1}{m_h} \right) \left(1 - \frac{N-n}{\chi_{1-\alpha/2, N-n}^2} \right) s_{dw}^2 \right]^2}$$

Bland-Altman Plot and Analysis

Step 2. Compute *LME* and *RME* as follows.

$$LME = \sqrt{\frac{z_{\alpha/2}^2 S_d^2}{n} + z_{\beta/2}^2 \left(\sqrt{u} - \sqrt{S_d^2} \right)^2}$$

$$RME = \sqrt{\frac{z_{\alpha/2}^2 S_d^2}{n} + z_{\beta/2}^2 \left(\sqrt{l} - \sqrt{S_d^2} \right)^2}$$

Step 3. Compute the MOVER confidence intervals as follows.

The $(1 - \alpha)\%$ MOVER confidence interval for the lower limit of agreement is

$$LoA_{lower} - LME, LoA_{lower} + RME$$

The $(1 - \alpha)\%$ MOVER confidence interval for the upper limit of agreement is

$$LoA_{upper} - RME, LoA_{upper} + LME$$

Limits of Agreement Calculation – Individual Differences.

This approach is mentioned in Olofsen, Dahan, Borsboom, and Drummond (in press as of July 2014). In this approach, the bias is measured by the grand mean of the individual differences rather than by the mean of the subject differences. The authors give situations in which this approach might be better. They outline the rather complicated calculations for this method. We refer you to their article for the details of our implementation of the method.

Example 1 – Bland-Altman Plots and Reports

This section presents an example of how to generate a Bland-Altman plot. In this example, two measurements were made on each of 100 subjects. The first measurement was made by a lengthy, invasive method and the second measurement was made by a second, much less invasive method. The data are in the **Bland-Altman** dataset. The engineers wish to analyze whether the two methods agree.

Setup

To run this example, complete the following steps:

1 Open the Bland-Altman example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Bland-Altman** and click **OK**.

2 Specify the Bland-Altman Plot and Analysis procedure options

- Find and open the **Bland-Altman Plot and Analysis** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

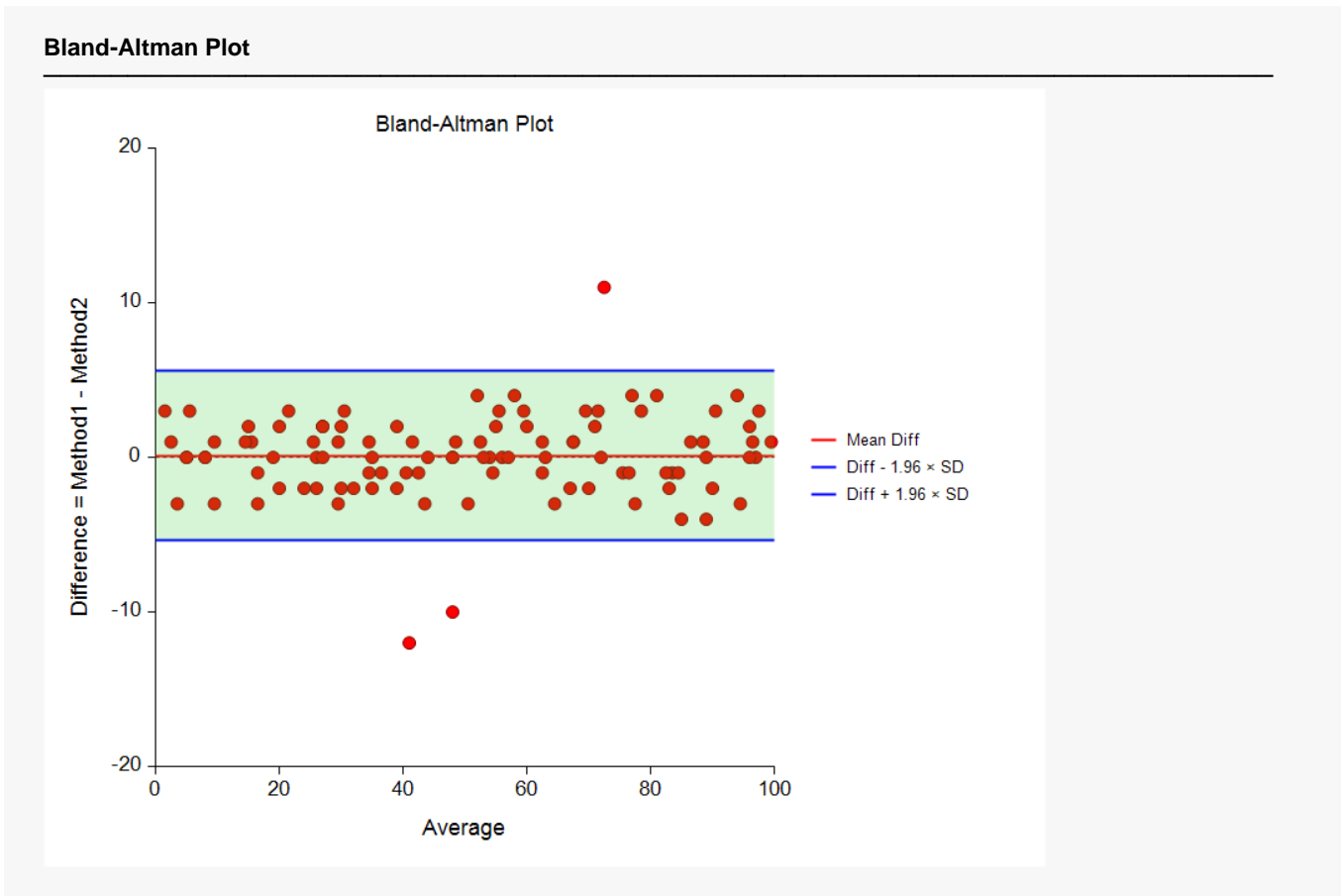
Variables Tab

Data Input Type	1. One row per subject with only one replicate for each method.
Method 1 Variable.....	Method1
Method 2 Variable.....	Method2

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Bland-Altman Plot



This is an example of the Bland-Altman plot. The average of the two measurements is plotted along the horizontal axis and the difference between the two methods is plotted along the vertical axis.

Look for trends, patterns, and anomalies in this plot. A few outliers are to be expected. They do not need to be removed.

Descriptive Statistics

Descriptive Statistics

Variable	Count	Mean	Standard Deviation	95.0% LCL of Mean	95.0% UCL of Mean
Method1	100	50.72	28.30893	45.10289	56.3371
Method2	100	50.62	28.07701	45.04891	56.19109
Difference	100	0.1	2.787055	-0.4530122	0.6530122

Correlation

Correlation Coefficient = 0.995147

This report provides basic descriptive statistics and confidence intervals for the two variables and their difference.

Bland-Altman Analysis

Bland-Altman Analysis: Bias and Limits of Agreement for Method1 and Method2

Limits of Agreement = $\text{Diff} \pm 1.96 \times (\text{Std Dev of Difference})$

Parameter	Count	Value	Standard Deviation	95.0% LCL of Value	95.0% UCL of Value
Bias (Difference)	100	0.1	2.787055	-0.4530122	0.6530122
Lower Limit of Agreement	100	-5.362628	0.4778968	-6.310879	-4.414377
Upper Limit of Agreement	100	5.562628	0.4778968	4.614377	6.510879

The report provides the bias (mean difference) and the limits of agreement in the Values column. Also included are 95% confidence intervals for each quantity.

*The standard deviations for the lower and upper limits of agreement are actually standard errors.

Test of Normality of Differences

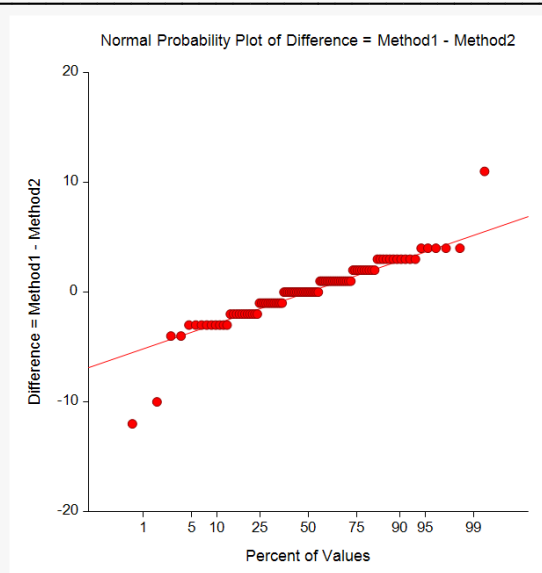
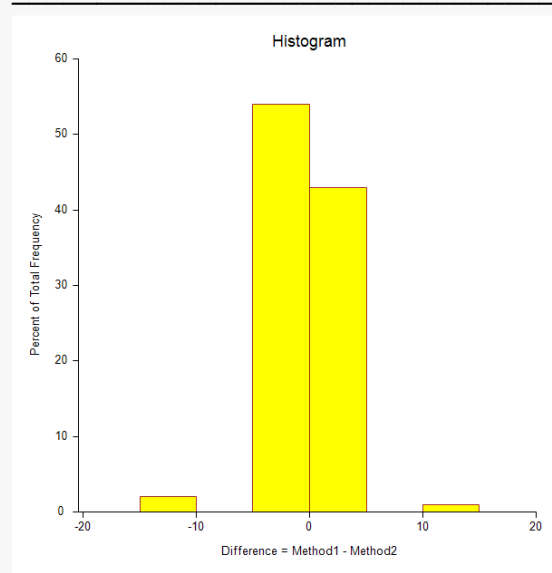
Test of Normality of Differences Assumption

Assumption	Value	Prob Level	Decision ($\alpha = 0.050$)
Shapiro-Wilk	0.893	0.0000	Reject normality

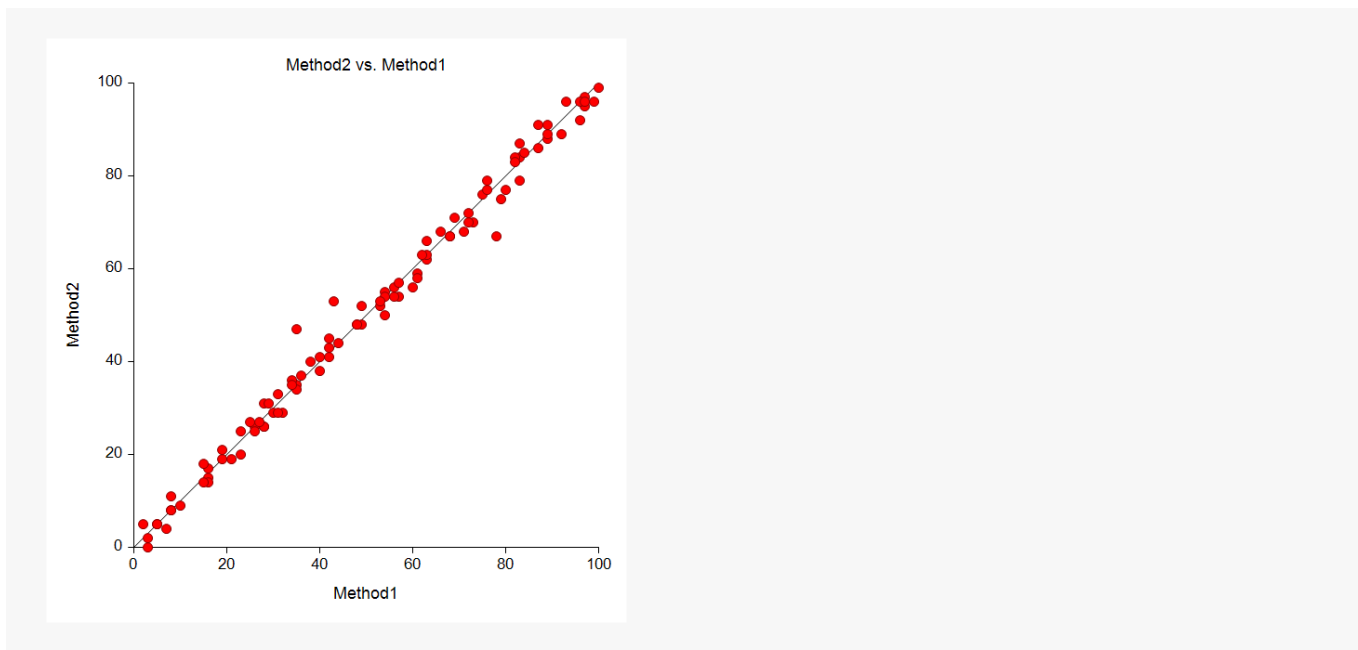
This section reports the results of a diagnostic test to determine if the differences are normal. In this case, they are not, probably because of the outliers that were found.

Evaluation of Assumptions Plots

Evaluation of Assumptions Plots



Bland-Altman Plot and Analysis



Histogram

The histogram provides a general idea of the distribution of the differences.

Normal Probability Plot

If the observations fall along a straight line, this indicates the data follow a normal distribution.

Method1 vs Method2 Plot

This plot lets you see the relationship between the two methods. The 45-degree diagonal line is a reference line that shows perfect equality between the two variables.

Example 2 – Bland-Altman Plot with Confidence Intervals

This section shows the result of embellishing the standard Bland-Altman plot with confidence intervals for the three horizontal lines. The data are in the **Bland-Altman** dataset.

Setup

To run this example, complete the following steps:

1 Open the Bland-Altman example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Bland-Altman** and click **OK**.

2 Specify the Bland-Altman Plot and Analysis procedure options

- Find and open the **Bland-Altman Plot and Analysis** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Data Input Type **1. One row per subject with only one replicate for each method.**
 Method 1 Variable..... **Method1**
 Method 2 Variable..... **Method2**

Plots Tab

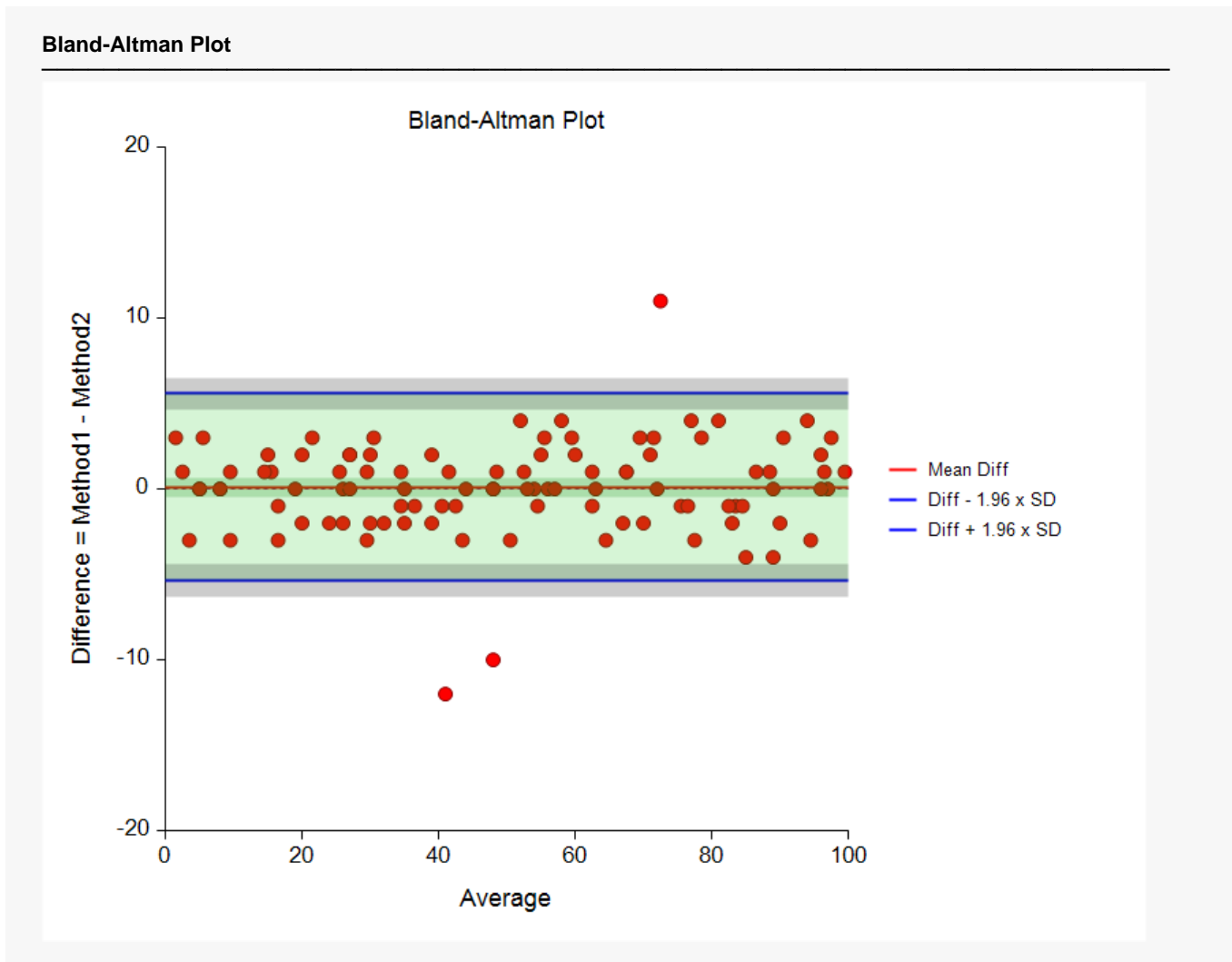
Bland-Altman Plot Format (*Click the Button under Data Input Type 1*)

Mean Difference Fill..... **Checked**
 Lower Limit Fill..... **Checked**
 Upper Limit Fill..... **Checked**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Bland-Altman Plot



This version of the Bland-Altman plot adds confidence intervals for the mean difference (darker green horizontal bar) and the agreement limits (gray bars). These give you a visual impression of the precision of these lines.

Example 3 – Bland-Altman Plot with Data Input Type = 2

This section presents an example of how to generate a Bland-Altman plot for Data Input Type 2. Suppose that 12 subjects are measured up to six times using each of two measurement devices: RV and IC. Although the data are entered on the database as RV1 – RV6 followed by IC1 – IC6, they were not always obtained in this order. Sometimes the RV measurements were made first, while other times the IC measurements were made first, depending on the flip of a coin. The data are in the **Bland-Altman – Data Input Type 2** dataset. The engineers wish to analyze whether the two methods are in complete agreement.

Setup

To run this example, complete the following steps:

1 Open the Bland-Altman2 example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Bland-Altman2** and click **OK**.

2 Specify the Bland-Altman Plot and Analysis procedure options

- Find and open the **Bland-Altman Plot and Analysis** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

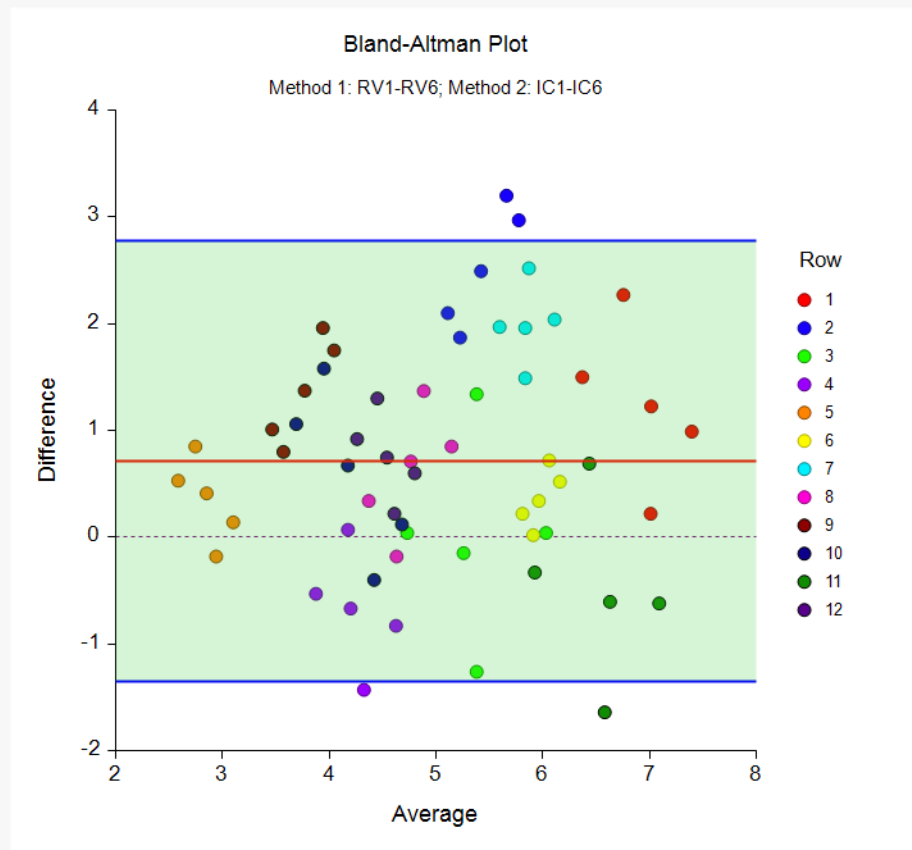
Data Input Type	2. One row per subject with multiple replicates for each method.
Method 1 Variables	RV1-RV6
Method 2 Variables	IC1-IC6

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Bland-Altman Plot

Bland-Altman Plot



Variables Analyzed

Method	Variables
1	RV1-RV6
2	IC1-IC6

This is an example of the Bland-Altman plot for the case when pairing does not matter. In this case, each subject is plotted using a different colored symbol. Since there measurements are not paired, it is somewhat arbitrary which values are plotted. We chose to plot five points per subject, no matter how many possible combinations of points where possible. The five points were generated as follows:

1. Determine the minimum, maximum, and average for the method 1 variables.
2. Determine the minimum, maximum, and average for the method 2 variables.
3. Compute four pairs using all combinations of the minimum and maximum values of each method. For example, pair min RV with max IC.
4. Compute one additional pair using the average RV and then average IC.
5. Compute the difference and average values of these five pairs and plot them.

You can scan the plot to learn what you can about the patterns in the data.

Descriptive Statistics

Descriptive Statistics

Method	Subject Count n	Observation Count N	Mean	Mean-Square Between-Subjects MSB = s^2	Mean-Square Within-Subjects MSE = $s^2[w]$
1	12	60	5.3895	1.805107	0.1072278
2	12	60	4.680264	1.612857	0.1378741
Difference	12	120	0.7092361	0.9126912	

Correlation between Methods 1 and 2 Subject Means = 0.734134

This report provides the means and mean squares of each set of variables. The mean-square between and within subjects were computed from a one-way analysis of variance on each variable.

Mean Difference between Methods

Mean Difference between Methods with Confidence Interval

Parameter	Subject Count	Value	Standard Deviation	95.0% LCL of Mean	95.0% UCL of Mean
Mean Difference	12	0.7092361	0.9553487	0.1022365	1.316236

This report provides the mean of the subject differences, their standard deviations, and confidence limits for the mean.

Limits of Agreement with Confidence Intervals using MOVER

Limits of Agreement with Confidence Intervals using MOVER

Limit of Agreement	Subject Count	Value	Standard Deviation	95.0% LCL of Value	95.0% UCL of Value
Lower	12	-1.352391	0.4563031	-2.699204	-0.6283661
Upper	12	2.770863	0.4563031	2.046838	4.117676

This report provides the limits of agreement (LoA) along with their confidence limits.

Example 4 – Bland-Altman Plot with Data Input Type = 3

This section presents an example of how to generate a Bland-Altman plot for Data Input Type 3. Suppose that 12 subjects are measured a variable number of times on each of two measurement devices: RV and IC. Each measurement pair is recorded on a row of the database. The data are in the **Bland-Altman – Data Input Type 3** dataset. The engineers wish to analyze whether the two methods are in complete agreement.

Setup

To run this example, complete the following steps:

1 Open the Bland-Altman3 example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Bland-Altman3** and click **OK**.

2 Specify the Bland-Altman Plot and Analysis procedure options

- Find and open the **Bland-Altman Plot and Analysis** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

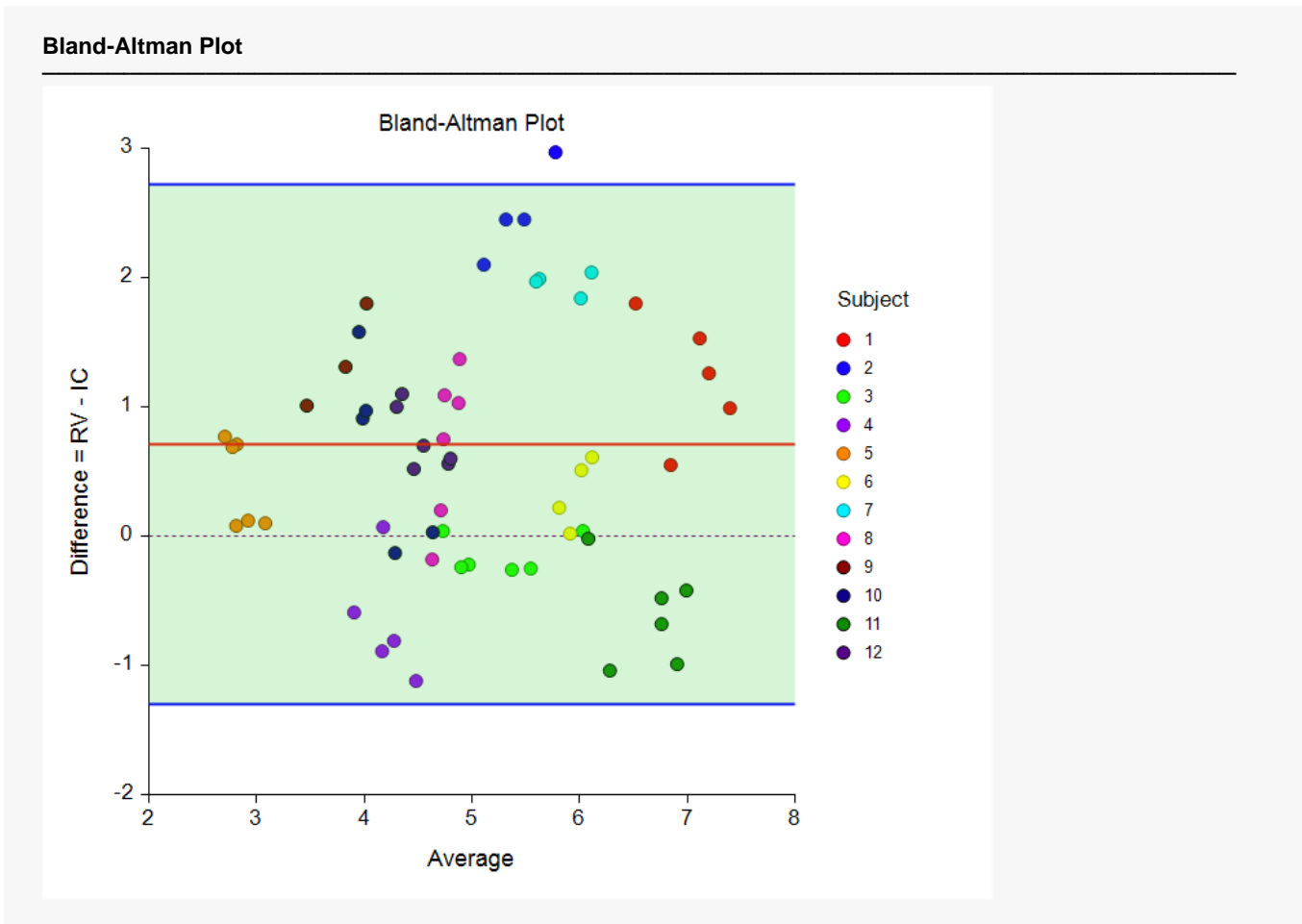
Variables Tab

Data Input Type	3. Multiple rows per subject with one replicate for each method per row.
Subject Variable.....	Subject
Method 1 Variable.....	RV
Method 2 Variable.....	IC

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Bland-Altman Plot



This is an example of the Bland-Altman plot for the case when there are multiple replicates per subject and pairing does matter. Each point represents a row of the dataset.

Look for outliers and changes in the vertical spread as you move your eye horizontally across the plot.

Descriptive Statistics

Descriptive Statistics							
Variable	Subject Count n	Observation Count N	Mean	Mean-Square Between-Subjects MSB	Mean-Square Within-Subjects MSE = $s^2[w]$	Variance Between-Subjects $s^2[I]$	Variance Between and Within s^2
RV	12	60	5.3895	9.066264	0.1072278	1.782619	1.889847
IC	12	60	4.680264	8.359395	0.1378741	0.9259933	1.063867
Difference	12	60	0.7092361	4.209086	0.170714	0.8768886	1.047603

Correlation Between Subject Means = 0.734134

This report provides the means, mean squares, and variances for each set of variables. The mean-square between and within subjects were computed from a one-way analysis of variance on each variable.

Mean Difference Between Methods

Mean Difference between Methods with Confidence Interval

Limits of Agreement = Diff \pm 1.96 \times (Std Dev of Difference)

Parameter	Subject Count	Value	Standard Deviation	95.0% LCL of Value	95.0% UCL of Value
Mean of Subject Differences	12	0.7092361	0.2757854	0.1687066	1.249766

This report provides the mean of the subject differences, their standard deviations, and confidence limits for the mean.

Limits of Agreement with Confidence Intervals using MOVER

Limits of Agreement for RV and IC with C.I. using MOVER

Limits of Agreement = Diff \pm 1.96 \times (Std Dev of Difference)

Parameter	Subject Count	Value	Standard Deviation	95.0% LCL of Value	95.0% UCL of Value
Lower Limit of Agreement	12	-1.296872	0.4643287	-2.662969	-0.5610639
Upper Limit of Agreement	12	2.715344	0.4643287	1.979536	4.081441

This report provides the limits of agreement (LoA) along with their confidence limits.

Variance and Standard Deviation Estimates for Difference

Variance Estimates for Difference between RV and IC

Limits of Agreement = Diff \pm 1.96 \times (Std Dev of Difference)

Bias Estimation Method	Variance of Mean Difference $s^2[B]$	Variance Between-Subjects $s^2[dl]$	Variance Within-Subject $s^2[dw]$	Variance Between + Within $s^2[d]$	Variance of Limits of Agreement $s^2[LoA]$
Subject Difference	0.0760576	0.8768886	0.170714	1.047603	0.2156011

Bland-Altman Plot and Analysis

Standard Deviation Estimates for Difference between RV and IC

 Limits of Agreement = Diff \pm 1.96 \times (Std Dev of Difference)

Bias Estimation Method	Std Dev of Mean Difference s[B]	Std Dev Between-Subjects s[dI]	Std Dev Within-Subject s[dw]	Std Dev Between + Within s[d]	Std Dev of Limits of Agreement s[LoA]
Subject Difference	0.2757854	0.9364233	0.4131755	1.023525	0.4643287

This report provides estimates for the variances (and standard deviations) of the quantities used in the calculations of the limits of agreement and their confidence intervals.