Chapter 238

Acceptance Sampling for Attributes

Introduction

The *Acceptance Sampling for Attributes* procedure is used to determine the number of items to be sampled from a lot to determine whether to accept or reject the lot. The number of items in the sample depend upon a number of parameters, including the lot size, the acceptable quality level (AQL), the desired producer's risk, the limiting quality level (LQL, sometimes called the rejectable quality level or lot tolerance percent/proportion defective), and the desired consumer's risk. This procedure permits the user to enter multiple values of any of these parameters to determine the sensitivity of the sample size to that parameter. When multiple values are entered for a parameter, a sample size curve is also produced. The cutoff value of acceptance, or acceptance number, is also given as part of the output.

In this procedure, the lot size can be assumed to be infinite (or continuous) and use the binomial distribution for calculations, or the lot can have a fixed size, whereupon the calculations are based on the hypergeometric distribution.

Technical details

Define *N* to be the lot size (possibly infinite), *n* as the (unknown) size of the sample to be drawn, and *c* to be the acceptance number (the highest number of nonconforming units for which the lot will still be accepted). Let *X* denote the number of nonconforming units in the sample. Let p_0 be the AQL, the highest proportion of nonconforming (defective) units for which the lot should still be accepted. Let α be the producer's risk, the probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the AQL. Let p_1 be the LQL, the proportion of nonconforming (defective) units above which the lot should be routinely rejected. Let β be the probability of accepting a lot with a proportion of nonconforming (defective) units that is above the LQL.

For a given *N*, p_0, α, p_1, and β, we desire to obtain an *n* and *c* such that

$$\Pr\{X \leq c|p_0\} \geq 1 - \alpha$$

and

$$\Pr\{X \leq c|p_1\} \leq \beta$$

If the lot size is finite, *n* and *c* should satisfy the hypergeometric distribution inequalities

$$H(c; N, M_0, n) \geq 1 - \alpha$$

and

$$H(c; N, M_1, n) \leq \beta$$

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where M_0 = [Np_0] and M_1 = [Np_1]. The hypergeometric probability of obtaining exactly *x* of *n* items with the characteristic of interest is calculated using

$$h(x;N,M,n) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

The cumulative hypergeometric distribution function is

$$H(x;N,M,n) = \sum_{j=0}^{x} h(j;N,M,n)$$

See Kenett and Zacks (2014) for more details.

If the lot size is infinite (or continuous), n and c should satisfy the binomial distribution inequalities

$$\sum_{j=0}^{c} \frac{n!}{j!\,(n-j)!} p_0^j\,(1-p_0)^{n-j} \geq 1-\alpha$$

and

$$\sum_{j=0}^{c} \frac{n!}{j!\,(n-j)!} p_1^j\,(1-p_1)^{n-j} \leq \beta$$

NCSS performs a search to determine an *n* and *c* value that meet the specified inequality requirements.

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Example 1 – Finding a Sample Size

Suppose a quality engineer is to receive a lot of 5,000 units. Because it is not feasible to inspect all 5,000 units, the engineer would like to examine an acceptance sample. It is determined that the acceptable quality level is 0.5%, or 0.005. The desired producer's risk (probability of rejecting a good lot) is 0.05. The limiting quality level (or lot tolerance percent defective or rejectable quality level) is 0.07. The desired consumer's risk (probability of accepting a bad lot) is 0.05. The engineer would like to know the number of units to randomly sample and inspect, as well as the acceptance number (the highest number of nonconforming units of the sample for which the lot should still be accepted).

Setup

To run this example, complete the following steps:

1 Specify the Acceptance Sampling for Attributes procedure options

- Find and open the **Acceptance Sampling for Attributes** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Sample Size**
Lot Size (N)	**5000**
Acceptable Quality Level	**0.005**
Producer's Risk (Alpha)	**0.05**
Limiting Quality Level	**0.07**
Consumer's Risk (Beta)	**0.05**
Approximation Cutoff	**10000**

2 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Output

Numeric Results

Lot Size (N) = 5000

Sample Size (n)	# Nonconforming c Accept Lot If # ≤	# Nonconforming c + 1 Reject Lot If # ≥	Acceptable Quality Level	Producer's Risk Target	Producer's Risk Actual	Limiting Quality Level	Consumer's Risk Target	Consumer's Risk Actual
66	1	2	0.0050	0.0500	0.0424	0.0700	0.05000	0.04860

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Sample Size (n)	The size of the sample drawn from the lot.
c	Acceptance Number. The largest number of nonconforming items from the sample of size n for which the lot should still be accepted.
c + 1	Rejection Number. The smallest number of nonconforming items from the sample of size n for which the lot should be rejected.
Acceptable Quality Level	AQL or P0. The highest proportion of nonconforming (defective) units for which the lot is still considered acceptable.
Producer's Risk	Alpha. The probability of rejecting a lot with a proportion of nonconforming (defective) units that is below the acceptable quality level. In short, it is the risk of rejecting a good lot.
Target Producer's Risk	The specified value entered by the user.
Actual Producer's Risk	The value of the producer's risk that is generated by the corresponding sample size. It is usually less than the target producer's risk.
Limiting Quality Level	LQL, P1, Rejectable Quality Level, or LTPD (for Lot Tolerance Percent/Proportion Defective). The proportion of nonconforming (defective) units above which the lot should be routinely rejected.
Consumer's Risk	Beta. The probability of accepting a lot with a proportion of nonconforming (defective) units that is above the limiting quality level. In short, it is the risk of accepting a bad lot.
Target Consumer's Risk	The specified value entered by the user.
Actual Consumer's Risk	The value of the consumer's risk that is generated by the corresponding sample size. It is usually less than the target consumer's risk.

Summary Statements

A sample size of 66 from a lot of size 5000 with an acceptance cutoff number (c) of 1, and based on an acceptable quality level of 0.0050 and a limiting quality level of 0.0700, has a producer's risk of 0.0424 and a consumer's risk of 0.04860. The target producer's risk was 0.0500. The target consumer's risk was 0.05000.

References

Chow, S. C., Shao, J., and Wang, H. 2008. Sample Size Calculations in Clinical Research, Second Edition. Chapman & Hall/CRC. Boca Raton, Florida.

Fleiss, J. L., Levin, B., and Paik, M.C. 2003. Statistical Methods for Rates and Proportions. Third Edition. John Wiley & Sons. New York.

Lachin, John M. 2000. Biostatistical Methods. John Wiley & Sons. New York.

Machin, D., Campbell, M., Fayers, P., and Pinol, A. 1997. Sample Size Tables for Clinical Studies, 2nd Edition. Blackwell Science. Malden, Mass.

Ryan, Thomas P. 2013. Sample Size Determination and Power. John Wiley & Sons. Hoboken, New Jersey.

Zar, Jerrold H. 2010. Biostatistical Analysis, 5th Edition. Prentice-Hall. Englewood Cliffs, New Jersey.

Kenett, Ron S. and Zacks, Shelemyahu 2014. Modern Industrial Statistics, 2nd Edition. John Wiley & Sons. West Sussex, United Kingdom.

This report shows that the needed sample size is 66. The lot should be accepted if the number of nonconforming units out of the sample of size 66 is 0 or 1. The lot should be rejected if the number of nonconforming units is 2 or greater.

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Example 2 – Finding Sample Sizes for Multiple Parameter Values

Continuing with Example 1, suppose the quality engineer would like to see how the sample size is affected by varying some of the parameters. The acceptable quality level is to stay at 0.005, but the engineer would like to look at Producer's Risk values between 0.01 and 0.1, Limiting Quality Level values between 0.05 and 0.1, and Consumer's Risk values between 0.05 and 0.2.

Setup

To run this example, complete the following steps:

1 Specify the Acceptance Sampling for Attributes procedure options

- Find and open the **Acceptance Sampling for Attributes** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Sample Size**
Lot Size (N)	**5000**
Acceptable Quality Level	**0.005**
Producer's Risk (Alpha)	**0.01 to 0.1 by 0.01**
Limiting Quality Level	**0.05 to 0.1 by 0.01**
Consumer's Risk (Beta)	**0.05 to 0.2 by 0.05**
Approximation Cutoff	**10000**

2 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

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Output

Numeric Results

Numeric Results

Lot Size (N) = 5000

Sample Size (n)	# Nonconforming c Accept Lot If # ≤	# Nonconforming c + 1 Reject Lot If # ≥	Acceptable Quality Level	Producer's Risk Target	Producer's Risk Actual	Limiting Quality Level	Consumer's Risk Target	Consumer's Risk Actual
152	3	4	0.0050	0.0100	0.0063	0.0500	0.05000	0.04871
152	3	4	0.0050	0.0200	0.0063	0.0500	0.05000	0.04871
123	2	3	0.0050	0.0300	0.0225	0.0500	0.05000	0.04945
123	2	3	0.0050	0.0400	0.0225	0.0500	0.05000	0.04945
123	2	3	0.0050	0.0500	0.0225	0.0500	0.05000	0.04945
123	2	3	0.0050	0.0600	0.0225	0.0500	0.05000	0.04945
123	2	3	0.0050	0.0700	0.0225	0.0500	0.05000	0.04945
93	1	2	0.0050	0.0800	0.0779	0.0500	0.05000	0.04853
93	1	2	0.0050	0.0900	0.0779	0.0500	0.05000	0.04853
93	1	2	0.0050	0.1000	0.0779	0.0500	0.05000	0.04853
126	3	4	0.0050	0.0100	0.0032	0.0600	0.05000	0.04968
103	2	3	0.0050	0.0200	0.0141	0.0600	0.05000	0.04784
103	2	3	0.0050	0.0300	0.0141	0.0600	0.05000	0.04784
103	2	3	0.0050	0.0400	0.0141	0.0600	0.05000	0.04784
103	2	3	0.0050	0.0500	0.0141	0.0600	0.05000	0.04784
77	1	2	0.0050	0.0600	0.0559	0.0600	0.05000	0.04924
77	1	2	0.0050	0.0700	0.0559	0.0600	0.05000	0.04924
77	1	2	0.0050	0.0800	0.0559	0.0600	0.05000	0.04924
77	1	2	0.0050	0.0900	0.0559	0.0600	0.05000	0.04924
77	1	2	0.0050	0.1000	0.0559	0.0600	0.05000	0.04924
88	2	3	0.0050	0.0100	0.0092	0.0700	0.05000	0.04800
88	2	3	0.0050	0.0200	0.0092	0.0700	0.05000	0.04800
88	2	3	0.0050	0.0300	0.0092	0.0700	0.05000	0.04800
88	2	3	0.0050	0.0400	0.0092	0.0700	0.05000	0.04800
66	1	2	0.0050	0.0500	0.0424	0.0700	0.05000	0.04860
66	1	2	0.0050	0.0600	0.0424	0.0700	0.05000	0.04860
66	1	2	0.0050	0.0700	0.0424	0.0700	0.05000	0.04860
66	1	2	0.0050	0.0800	0.0424	0.0700	0.05000	0.04860
66	1	2	0.0050	0.0900	0.0424	0.0700	0.05000	0.04860
66	1	2	0.0050	0.1000	0.0424	0.0700	0.05000	0.04860
77	2	3	0.0050	0.0100	0.0063	0.0800	0.05000	0.04735
77	2	3	0.0050	0.0200	0.0063	0.0800	0.05000	0.04735
77	2	3	0.0050	0.0300	0.0063	0.0800	0.05000	0.04735
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

Each line gives a sample size and acceptance number based on the corresponding combination of quality levels and risks.

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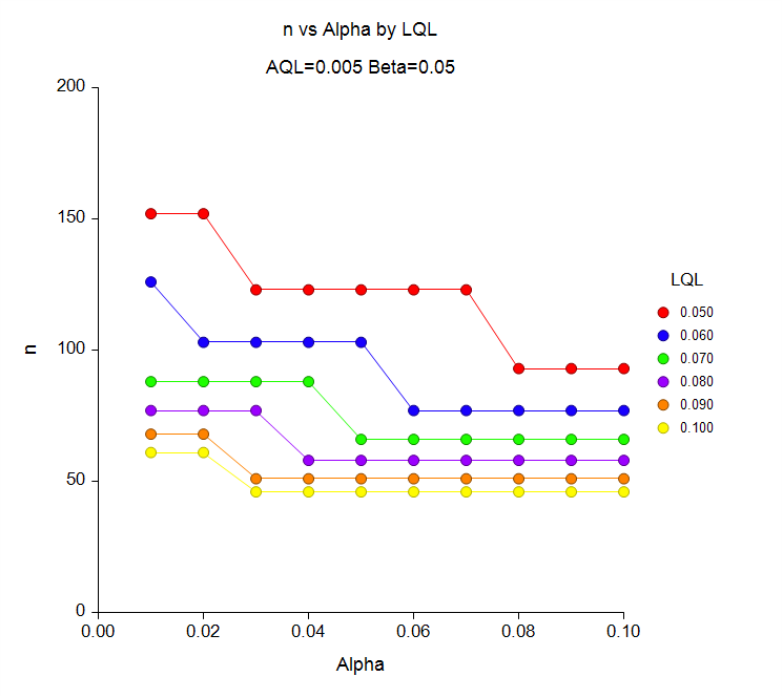
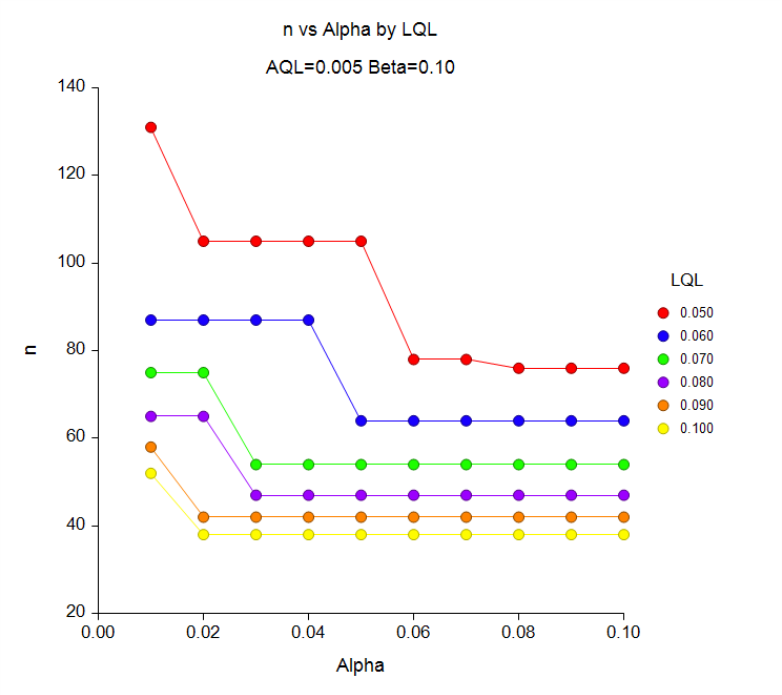
Chart Section

Chart Section





(More Charts Follow)

These plots show the effect of various combinations of parameters on the sample size needed.

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Example 3 – Validation of Sample Size Calculation using Kenett and Zacks (2013)

Kenett and Zacks (2013), pp. 260-261, give an example in which the lot size is 100, alpha and beta (producer's risk and consumer's risk) are both 0.05, the acceptable quality level is 0.01, and the limiting quality level is 0.05. The needed sample size is estimated to be 65, with a *c* of 1.

Setup

To run this example, complete the following steps:

1 Specify the Acceptance Sampling for Attributes procedure options

- Find and open the **Acceptance Sampling for Attributes** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 3(a-c)** settings files. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Sample Size**
Lot Size (N)	**100**
Acceptable Quality Level	**0.01**
Producer's Risk (Alpha)	**0.05**
Limiting Quality Level	**0.05**
Consumer's Risk (Beta)	**0.05**
Approximation Cutoff	**10000**

2 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Output

Numeric Results

Lot Size (N) = 100

	# Nonconforming							
Sample Size (n)	c Accept Lot If # ≤	c + 1 Reject Lot If # ≥	Acceptable Quality Level	Producer's Risk Target	Producer's Risk Actual	Limiting Quality Level	Consumer's Risk Target	Consumer's Risk Actual
65	1	2	0.0100	0.0500	0.0000	0.0500	0.05000	0.04952

The sample size of 65 and *c* value of 1 match that of Kenett and Zacks (2013).

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In a second example with acceptable quality level of 0.03, and a limiting quality level of 0.05, Kenett and Zacks (2013) estimate a required sample size of 92.

Numeric Results

Lot Size (N) = 100

Sample Size (n)	# Nonconforming: c Accept Lot If # ≤	# Nonconforming: c + 1 Reject Lot If # ≥	Acceptable Quality Level	Producer's Risk: Target	Producer's Risk: Actual	Limiting Quality Level	Consumer's Risk: Target	Consumer's Risk: Actual
92	3	4	0.0300	0.0500	0.0000	0.0500	0.05000	0.04990

NCSS also gives a sample size estimate of 92.

In a third example with acceptable quality level of 0.01, and a limiting quality level of 0.2, Kenett and Zacks (2013) estimate a required sample size of 20.

Numeric Results

Lot Size (N) = 100

Sample Size (n)	# Nonconforming: c Accept Lot If # ≤	# Nonconforming: c + 1 Reject Lot If # ≥	Acceptable Quality Level	Producer's Risk: Target	Producer's Risk: Actual	Limiting Quality Level	Consumer's Risk: Target	Consumer's Risk: Actual
20	1	2	0.0100	0.0500	0.0000	0.2000	0.05000	0.04985

NCSS also estimates a sample size estimate of 20 for this scenario.

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Example 4 – Validation of Sample Size Calculation using Guenther (1977)

Guenther (1977), pp. 7-8, gives an example in which the lot size is infinite (binomial calculations), alpha (producer's risk) is 0.05, beta (consumer's risk) is 0.1, the acceptable quality level is 0.05, and the limiting quality level is 0.2. The needed sample size is estimated to be 38, with an acceptance number (*c*) of 4.

Setup

To run this example, complete the following steps:

1 Specify the Acceptance Sampling for Attributes procedure options

- Find and open the **Acceptance Sampling for Attributes** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab	
Solve For	**Sample Size**
Lot Size (N)	**Infinite**
Acceptable Quality Level	**0.05**
Producer's Risk (Alpha)	**0.05**
Limiting Quality Level	**0.2**
Consumer's Risk (Beta)	**0.1**
Approximation Cutoff	**10000**

2 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Output

Numeric Results

Lot Size (N) = Infinite

Sample Size (n)	# Nonconforming: c Accept Lot If # ≤	# Nonconforming: c + 1 Reject Lot If # ≥	Acceptable Quality Level	Producer's Risk: Target	Producer's Risk: Actual	Limiting Quality Level	Consumer's Risk: Target	Consumer's Risk: Actual
38	4	5	0.0500	0.0500	0.0397	0.2000	0.10000	0.09857

The sample size of 38 and *c* value of 4 match those of Guenther (1977).

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